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SIMULATION OF SOUND ABSORPTION IN 2D THIN ELEMENTS USING A COUPLED BEM/TBEM FORMULATION IN THE PRESENCE OF 3D SOURCES

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ABSTRACT

Modeling sound propagation in complex environments can be a difficult task. In realistic applications the boundaries of scatterers present in the propagation domain can be partially absorbing, and this must be accounted for in the numerical models. This paper addresses the use of a frequency-domain Dual-BEM (BEM/TBEM) formulation to model the propagation of sound generated by fixed and moving point loads in 2.5D configurations, in the presence of very thin elements with partially absorbing surfaces. The proposed approach is based on the concept of impedance boundary conditions and is applied in conjunction with a Dual-BEM approach, thereby allowing the definition of models in which only very compact descriptions of the propagation domain are required. Since a 2.5D formulation is used, 3D responses can be computed as a discrete summation of simpler 2D solutions. The formulation of the numerical methods used here (BEM, TBEM and Dual-BEM) are described, together with the strategy devised by the authors to incorporate sound absorption. A numerical application involving fixed and moving 3D sources is described to illustrate the applicability and usefulness of the proposed approaches.

Key words: Acoustic Wave Propagation, Sound Absorption, Thin Elements / Heterogeneities, Dual BEM/TBEM Formulation, Moving Sources, 3D Sources.

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Introduction

Some of the first applications of numerical methods to solve acoustics were centered on the use of finite differences or finite elements and the solution was sought in either the time or the frequency domain. In the 1970s the boundary element method became established as a consolidated method and with it new solution strategies were developed. The works by Marburg and Nolte [1] and Cheng and Cheng [2], and the reference book by Jensen et al [3] give a good general overview of the developments in this field.

We are focusing on the boundary element method (BEM) here, for which many references can be found related to its application in the time and the frequency domains (see, for example, [4-6]). Perhaps the most important advantage of this method is that it can easily account for infinite or semi-infinite domains, automatically satisfying the far-field radiation conditions. As it is based on fundamental solutions defined for infinite spaces, the method can be successfully used to model scenarios of outdoor sound propagation in which the propagation media may be described as unbounded domains.

However, the BEM can pose problems when in the context of specific system geometries. It is well known that the method cannot be directly applied to problems which involve very thin objects such as acoustic screens because the establishment of the boundary integral equations on both sides of the object leads to a singular equation system. However, special solution strategies can be adopted that can avoid this problem. The so-called Dual-BEM formulation is one such strategy (A. Mendes and Tadeu [7]).

Despite the large number of works on generic scenarios incorporating thin bodies with surface absorption, very few deal with the specific case in which acoustic waves generated by a point load propagate in a geometry that remains constant in one direction (the so-called 2.5D case). Even fewer account for the presence of a possible moving load travelling in the analysis scenario.

In this paper the authors address the use of a frequency-domain BEM formulation to model sound propagation in 2.5D configurations incorporating very thin elements with partially absorbing boundaries. In the sections that follow, the generic mathematical formulation of the problem is first presented, defining the solutions for both stationary and moving loads in infinite and semi infinite domains. Then, the numerical methods used here (BEM and TBEM) and their coupling procedure are described. The strategy devised to incorporate sound absorption is then explained. Finally, a methodology to obtain time-domain responses from frequency-domain calculations is described, followed by a numerical application to illustrate the applicability and usefulness of the proposed approach.

3D Incident Field

Fixed 3D Sources

Consider an irregular two-dimensional cylindrical inclusion, placed parallel to the z axis, bounded by a surface, S, and submerged in a spatially uniform fluid medium of density ρ, where the pressure waves propagate at velocity c. This system is subjected to a pressure point source placed at $(x_s, y_s\ z_s)$. The incident pressure field generated by this source can be expressed as

$$\hat{p}_{inc}(x,y,z,x_s,y_s,z_s,\omega)=\frac{e^{i\frac{\omega}{c}(ct-r_{000})}}{r_{000}} \tag{1}$$

in which ω is the oscillating frequency, $i=\sqrt{-1}$ and

$$r_{000}=\sqrt{(x-x_s)^2+(y-y_s)^2+(z-z_s)^2}\ .$$

As the geometry of the problem does not change in the z direction, the application of a spatial Fourier-transformation to Eq. (1) in that direction gives a line incident pressure field, whose amplitude varies sinusoidally in the third dimension (z),

$$p_{\text{inc}}(x,y,x_s,y_s,k_z,\omega)=\frac{-i}{2}H_0\left(k_c r_0\right)e^{-ik_z z}, \tag{2}$$

in which $H_n(\ldots)$ are second kind Hankel functions of the order n, $k_c=\sqrt{\frac{\omega^2}{c^2}-k_z^2}$, with $\operatorname{Im}(k_c)<0$, $r_0=\sqrt{(x-x_s)^2+(y-y_s)^2}$, where k_z is the axial wavenumber.

Assuming the presence of an infinite set of equally-spaced sources in the z direction, the former three-dimensional incident field can be written as:

$$\hat{p}_{inc}(x,y,z,x_s,y_s,\omega)=\frac{2\pi}{L_{vs}}\sum_{m=-\infty}^{\infty}p_{inc}\left(x,y,x_s,y_s,k_z,\omega\right)e^{-ik_{zm}z}, \tag{3}$$

where L_{vs} is the spatial source interval, and $k_{zm}=\frac{2\pi}{L_{vs}}m$.

This equation converges and can be approximated by a finite sum of terms. The distance L_{vs} needs to be large enough to avoid spatial contamination. The 3D pressure field can therefore be computed as the pressure irradiated by a sum of harmonic (steady-state) line loads.

Moving 3D Sources

Assuming that a moving source passes the cross-section $z = 0.0\,\mathrm{m}$ at the instant $t = 0.0\,\mathrm{s}$, with velocity c_m, the source spectrum in the frequency - wavenumber domain is obtained by means of a double Fourier transform with respect to the z direction and time, which results in the following incident field

$$p_{\mathrm{inc}}(x, y, x_s, y_s, k_z, \omega) = \frac{-\mathrm{i}}{2} H_0\left(k_c r_0\right) \mathrm{e}^{-\mathrm{i}k_z z}, \quad (4)$$

with $k_z = \dfrac{\omega}{c_m}$. The pressures can be retrieved back to the time domain by means of the application of an inverse Fourier transform.

Boundary Integral Formulations

Boundary element formulation

The classical boundary integral equation can be derived from the Helmholtz equation in the frequency domain by applying the reciprocity theorem, leading to:

$$\begin{aligned} b\, p(x_0, y_0, k_z, \omega) &= \int_S q(x, y, \mathbf{n}_{n1}, k_z, \omega)\, G(x, y, x_0, y_0, k_z, \omega)\, \mathrm{d}s \\ &- \int_S H(x, y, \mathbf{n}_{n1}, x_0, y_0, k_z, \omega)\, p(x, y, k_z, \omega)\, \mathrm{d}s + p_{\mathrm{inc}}(x_0, y_0, x_s, y_s, k_z, \omega) \end{aligned} \quad (5)$$

where G and H represent the Green's functions for the pressure (p) and pressure gradient (q) at a point (x, y) on the boundary S due to a virtual point pressure source at a collocation point (x_0, y_0). $\mathbf{n}_{n1}$ represents the unit outward normal along the boundary S, at (x, y), defined by the vector $\mathbf{n}_{n1} = (\cos\theta_{n1}, \sin\theta_{n1})$. The factor b takes the value $1/2$ if $(x_0, y_0) \in S$ and 1 otherwise.

The Green's functions for pressure and pressure gradients in an unbounded medium, in Cartesian co-ordinates, can be given by:

$$G(x, y, x_0, y_0, k_z, \omega) = -\frac{\mathrm{i}}{4} H_0\left(k_c r_{01}\right), \qquad H\left(x, y, \mathbf{n}_{n1}, x_0, y_0, k_z, \omega\right) = \frac{\mathrm{i}}{4} k_c H_1\left(k_c r_{01}\right) \frac{\partial r_{01}}{\partial \mathbf{n}_{n1}} \quad (6)$$

with $r_{01} = \sqrt{(x - x_0)^2 + (y - y_0)^2}$.

Traction boundary element formulation

The traction boundary integral equation can be derived by applying the gradient operator to the boundary integral equation (5), which can be seen as assuming the existence of dipole pressure sources. When the boundary of the inclusion is loaded with dipoles (dynamic doublets) the required integral equations can be expressed as:

$$a\, p(x_0,y_0,k_z,\omega)+b\, q(x_0,y_0,\mathbf{n}_{n1},k_z,\omega)=\int_S q(x,y,\mathbf{n}_{n1},k_z,\omega)\overline{G}(x,y,\mathbf{n}_{n2},x_0,y_0,k_z,\omega)\,\mathrm{ds}$$
$$-\int_S \overline{H}(x,y,\mathbf{n}_{n1},\mathbf{n}_{n2},x_0,y_0,k_z,\omega)\,p(x,y,k_z,\omega)\,\mathrm{ds}+\overline{p}_{inc}(x_0,y_0,\mathbf{n}_{n2},x_s,y_s,k_z,\omega) \quad (7)$$

The Green's functions $\overline{G}$ and $\overline{H}$ are defined by applying the traction operator to G and H, which can be seen as the derivatives of these former Green's functions, in order to obtain pressure gradients. In these equations, $\mathbf{n}_{n2}$ is the unit outward normal to the boundary S at the collocation points (x_0,y_0), defined by the vector $\mathbf{n}_{n2}=(\cos\theta_{n2},\sin\theta_{n2})$. In this equation, the factor a is null for piecewise straight boundary elements.

The required two-dimensional Green's functions for an unbounded space are now defined as:

$$\overline{G}(x,y,\mathbf{n}_{n2},x_0,y_0,k_z,\omega)=\frac{\mathrm{i}}{4}k_c H_1\left(k_c\, r_{01}\right)\frac{\partial r_{01}}{\partial \mathbf{n}_{n2}}$$

$$\overline{H}(x,y,\mathbf{n}_{n1},\mathbf{n}_{n2},x_0,y_0,k_z,\omega)=$$
$$\frac{\mathrm{i}}{4}k_c\left\{-k_c H_2\left(k_c\, r_{01}\right)\left[\left(\frac{\partial r_{01}}{\partial x}\right)^2\frac{\partial x}{\partial \mathbf{n}_{n1}}+\frac{\partial r_{01}}{\partial x}\frac{\partial r_{01}}{\partial y}\frac{\partial y}{\partial \mathbf{n}_{n1}}\right]+\frac{H_1\left(k_c\, r_{01}\right)}{r_{01}}\left[\frac{\partial x}{\partial \mathbf{n}_{n1}}\right]\right\}\frac{\partial x}{\partial \mathbf{n}_{n2}}+ \quad (8)$$
$$\frac{\mathrm{i}}{4}k_c\left\{-k_c H_2\left(k_c\, r_{01}\right)\left[\frac{\partial r_{01}}{\partial x}\frac{\partial r_{01}}{\partial y}\frac{\partial x}{\partial \mathbf{n}_{n1}}+\left(\frac{\partial r_{01}}{\partial y}\right)^2\frac{\partial y}{\partial \mathbf{n}_{n1}}\right]+\frac{H_1\left(k_c\, r_{01}\right)}{r_{01}}\left[\frac{\partial y}{\partial \mathbf{n}_{n1}}\right]\right\}\frac{\partial y}{\partial \mathbf{n}_{n2}}$$

In equation (7) the incident field is computed as

$$\overline{p}_{inc}(x_0,y_0,\mathbf{n}_{n2},x_s,y_s,k_z,\omega)=\frac{\mathrm{i}}{2}k_c H_1\left(k_c r_0\right)\left(\frac{\partial r_0}{\partial x}\frac{\partial x}{\partial \mathbf{n}_{n2}}+\frac{\partial r_0}{\partial y}\frac{\partial y}{\partial \mathbf{n}_{n2}}\right). \quad (9)$$

Coupling the TBEM and the BEM formulations

The BEM formulation cannot be used to calculate the scattering field when the inclusion is thin because it degenerates. The coupling between the TBEM and the BEM formulations is used to overcome this. The BEM and TBEM formulations are combined on opposite collocation points. Part of the boundary surface of the thin inclusion is loaded with monopole loads (BEM formulation, eq.(5)), while the remaining part is loaded with dipoles (TBEM formulation, eq.(7)).

Sound absorption simulation

The sound absorption is simulated by prescribing boundary conditions that relate the pressure and the velocity at each collocation point. This can be viewed as a Robin boundary condition (impedance boundary condition), that is

$$q(x,y,\mathbf{n}_{n1},k_z,\omega) = -\mathrm{i}\omega\rho\frac{1}{Z(\omega)}p(x,y,k_z,\omega) \tag{10}$$

Thus the following equations are defined for each collocation point of the host medium's boundary,

$$b\,p(x_0,y_0,k_z,\omega) = \int_S q(x,y,\mathbf{n}_{n1},k_z,\omega)\begin{bmatrix} G(x,y,x_0,y_0,k_z,\omega) + \\ \dfrac{Z(\omega)}{\mathrm{i}\rho\omega}H(x,y,\mathbf{n}_{n1},x_0,y_0,k_z,\omega)\end{bmatrix}\mathrm{d}s \quad \text{and} \tag{11}$$
$$+\,p_{\mathrm{inc}}(x_0,y_0,x_s,y_s,k_z,\omega)$$

$$a\,p(x_0,y_0,k_z,\omega) + b\,q(x_0,y_0,\mathbf{n}_{n1},k_z,\omega) =$$
$$\int_S q(x,y,n_{n1},k_z,\omega)\begin{bmatrix}\overline{G}(x,y,\mathbf{n}_{n2},x_0,y_0,k_z,\omega) + \\ \dfrac{Z(\omega)}{\mathrm{i}\rho\omega}\overline{H}(x,y,\mathbf{n}_{n1},\mathbf{n}_{n2},x_0,y_0,k_z,\omega)\end{bmatrix}ds + \overline{p}_{inc}(x_0,y_0,\mathbf{n}_{n2},x_s,y_s,k_z,\omega), \tag{12}$$

when applying a BEM and TBEM formulation, respectively.

In this paper the impedance $Z(\omega)$ is expressed by the ratio between the pressure and the velocity and the absorption coefficient α.

System of equations

The global solution is found by solving equations (11-12). This requires the discretization of the interface S, the boundary of inclusion. In this analysis the interface is discretised with N straight constant boundary elements. The required integrations in these equations are evaluated using a Gaussian quadrature scheme when they are not performed along the loaded element. For the loaded element, the existing singular and hypersingular integrands of the Green's functions are calculated analytically.

Numerical Application

The applicability and usefulness of the proposed approach are illustrated by simulating the wave propagation around thin acoustic screens hosted in a fluid medium geometry: a corner composed of two perpendicular flat surfaces, one simulating the horizontal ground surface and the other representing the façade of a tall building. To avoid the discretization of these rigid interfaces the Green's functions and the incident pressure need to be rewritten in a way that satisfies null normal velocities at those boundaries. This can be accomplished by adding the pressure field generated by the real source to that produced by virtual sources (image sources).

A 3D fixed and a 3D moving pressure source are simulated next. The 3D fixed source is placed at $x = 4.05\,\mathrm{m}$, $y = 2.0\,\mathrm{m}$ and $z = 0.0\,\mathrm{m}$. The host acoustic medium is air.

Pressure computations are performed in the frequency range of $[5.0,\ 1280.0\,\mathrm{Hz}]$ with a frequency step of $5.0\,\mathrm{Hz}$ over fine grids of receivers placed over three planes corresponding to: $x = 0.125\,\mathrm{m}$, $y = 0.125\,\mathrm{m}$ and $z = 0.0\,\mathrm{m}$. The receivers were spaced at equal intervals of $0.125\,\mathrm{m}$ in the x, y and z directions.

The 3D moving source travels along the z direction, at $x = 4.05\,\mathrm{m}$, $y = 2.0\,\mathrm{m}$, with velocities of $c_m = 50.0\,\mathrm{m/s}$ or $c_m = 400.0\,\mathrm{m/s}$. Pressure computations are performed in the frequency range of $[1.0,\ 1280.0\,\mathrm{Hz}]$ with a frequency step of $1.0\,\mathrm{Hz}$ at the grids of receivers described above.

Time responses were then obtained by applying an inverse Fourier transformation with the source temporal variation reproducing a Ricker pulse with a characteristic frequency of 350 Hz.

The effect of the absorption in the acoustic wavefield is illustrate by assuming that the surface of the thin inclusion (barrier or screen) has a constant sound absorption coefficient at all frequencies equal to $\alpha = 0.7$ while the remaining flat boundaries are rigid. This absorption is ascribed to the side of the barrier/screen facing the source.

3D Fixed Source

Figure 1 contains the time domain snapshots showing the pressure wavefield at different time instants ($t = 20.9\,\text{ms}$ and $t = 27.8\,\text{ms}$). At each instant two plots are displayed for each of the cases described above. The right plot corresponds to the rigid screens while the left one shows the results obtained when the absorption coefficient, $\alpha = 0.7$, is ascribed to the barrier/screen on the side of the source. In all snapshots, when the barriers or screens allow some absorption the wave amplitude suffers attenuation each time the waves reach them.

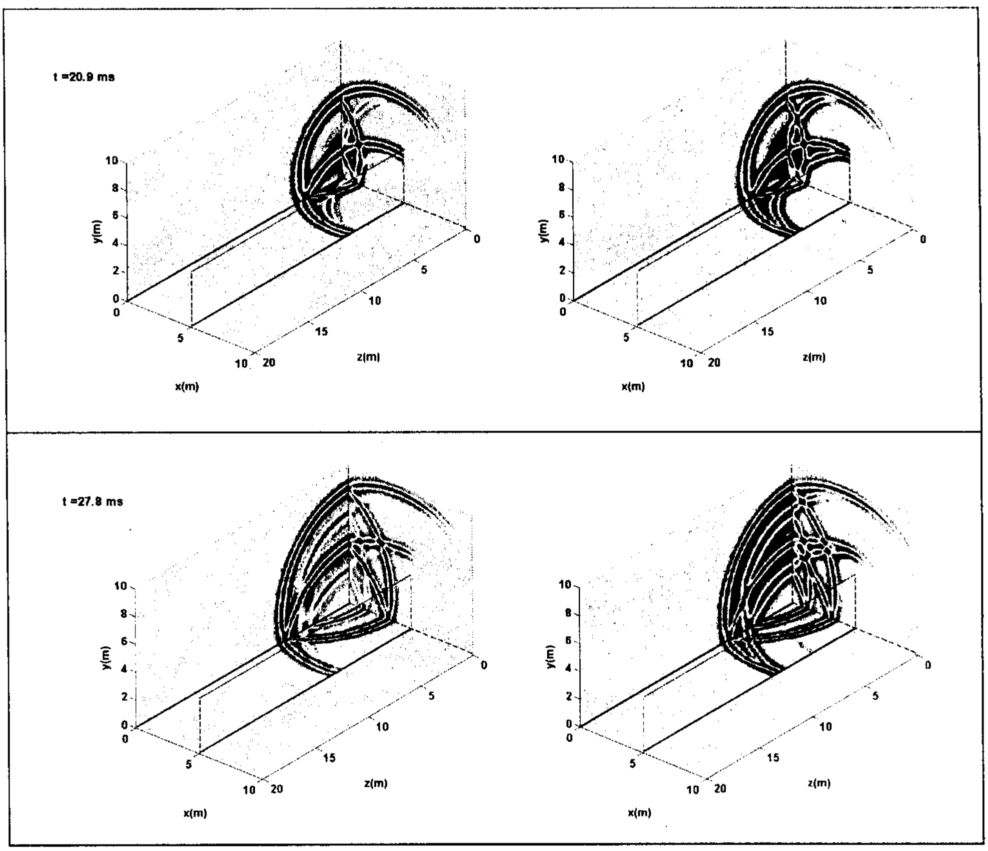
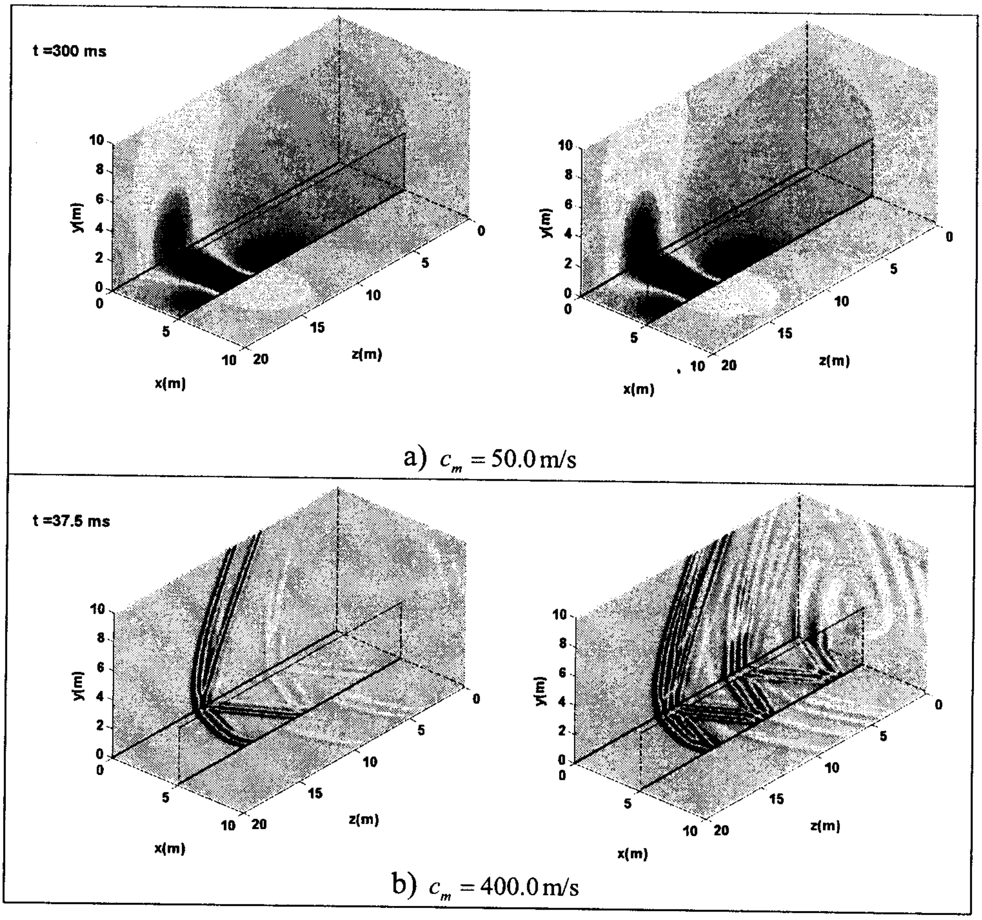


Figure 1: Fixed 3D Source: pressure responses.

3D Moving Source

Figure 2 displays time domain snapshots showing the pressure wavefield when moving sources travel along the z direction. These moving sources travel from

$z=-\infty$ to $z=+\infty$ at constant velocities $c_m = 50.0\,\text{m/s}$ and $c_m = 400.0\,\text{m/s}$. They pass at $t = 0.0\,\text{ms}$ by the plane $z = 0.0\,\text{m}$. Since the generated pressure field is constant as the moving source travels, only one snapshot is displayed. This corresponds to its passage at $z = 15.0\,\text{m}$, that is $t = 300.0\,\text{ms}$ and $t = 37.5\,\text{ms}$, in the presence of sources travelling at $c_m = 50.0\,\text{m/s}$ and $c_m = 400.0\,\text{m/s}$, respectively.



a) $c_m = 50.0\,\text{m/s}$

b) $c_m = 400.0\,\text{m/s}$

Figure 2: Moving Source: a) $c_m = 50.0\,\text{m/s}$; b) $c_m = 400.0\,\text{m/s}$.

$c_m = 50.0\,\text{m/s}$ corresponds to a travelling velocity slower than the pressure waves velocity. The pressure field ahead of the load oscillates between positive and negative values. It is visible a light pressure field that remains behind the load for some time.

When the source moves with a velocity $c_m = 400.0\,\text{m/s}$, the pressure wave-field forms a cone that moves with the source because the load is moving faster than the sound-wave speed of the medium.

Comparing the results for all cases when the moving velocity is $c_m = 50.0\,\text{m/s}$, the attribution of absorption in the barrier/screen does not seem to change the amplitude of the pressure field significantly. By contrast, when the velocity increases to $c_m = 400.0\,\text{m/s}$ the pressure field diminishes markedly in the presence of absorption.

Conclusions

In this paper the authors proposed a numerical formulation to allow modelling acoustic wave propagation in the presence of two-dimensional thin elements with absorbing surfaces when the medium is perturbed by fixed or moving 3D sources. In the proposed approach the concept of impedance boundary condition is used together with a Dual-BEM approach to allow the definition of models in which only very compact descriptions of the propagation domain are required. Since a 2.5D formulation is used, the 3D responses are defined as a discrete summation of simpler 2D solutions. Absorbing boundary conditions are incorporated by defining impedance coefficients defined by taking into account the pressure and the velocity of sound waves travelling directly from the source to each nodal boundary point, and by assuming a certain reflection coefficient for those incidence waves.

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