# ASSESSMENT OF DIFFERENT CORRELATION MODELS FOR ESTIMATING SOLAR IRRADIATION ON A NON-HORIZONTAL SURFACE

J. S. Carlos Centre of Materials and Building Technologies University of Beira Interior, Covilhã, Portugal

#### H. Corvacho

Faculty of Engineering of the University of Porto (FEUP), Civil Engineering Department, Building Division, Laboratory of Building Physics, Porto, Portugal

#### **ABSTRACT**

Global solar radiation on horizontal surface is often the only radiation registered data available for a specific location. In this paper, values of solar irradiation are estimated and then compared to measured values, on a vertical south-facing surface, in Covilhã, Portugal using 14 different (kt, kd) correlation models. A very acceptable agreement between estimated and measured values is achieved.

Key words: Clearness Index, Global Solar Irradiation, Tilted Surface, Diffuse Models.

# <u>Introduction</u>

Knowledge of the available solar energy is of extreme importance when passive solar systems are to be analyzed since their performance is highly dependent on that availability. These systems can have any position from horizontal to vertical and any orientation, although, it is a well-known fact that for the northern hemisphere, the orientation towards southern quadrant is the best, when solar heating is required. Normally, only global solar radiation on horizontal surface is measured in public meteorological weather stations. This being insufficient to estimate the solar radiation on any other surface, the concept of clearness index (kt) is then used. This represents

0146-6518/03/175-186, 2013 Copyright©2013 IAHS the percentage of solar radiation incident at the top of the atmosphere that reaches the Earth's surface. The kt index is defined as the "ratio of terrestrial over extra-terrestrial radiation, being the radiation which would be incident on the same horizontal surface in the absence of any atmosphere" as stated by Collares-Pereira and Rabl (1). The index kt is also defined as "the ratio of beam plus diffuse radiation on a horizontal surface to the extra-terrestrial radiation on a horizontal surface during the same hour" (2, 3).

Since the global solar radiation on a horizontal surface is composed by the direct and diffuse components in different proportions, a larger or smaller value of the global solar radiation leads to different relationships between these two components. The kd index (or diffuse fraction) is the percentage of diffuse radiation in the global solar radiation. Correlations between kt and kd indexes were established by previous works and then tested to estimate the values of kd, for given values of kt, allowing thus the determination of the direct and diffuse components of the global solar radiation and the estimation of the solar radiation on a non-horizontal surface. In the technical reference literature one can find a significant number of proposed (kt, kd) correlation models which confirms the complexity of the task.

Using a large number of data, based upon five different U. S. cities, Erbs et al (2) proposed a model to estimate the hourly diffuse radiation on a horizontal surface based on the hourly global solar radiation on a horizontal surface. The data recorded in Highett, Australia were found to agree with an accuracy of about 5% with the proposed hourly correlation. Lam and Danny (4) developed a correlation model which can predict the direct and diffuse components from measured global solar radiation for Hong Kong. The hourly global and diffuse radiation data measured during a four-year period (1991 – 1994) were used to develop the model for three sky condition periods (whole year, cooling season and heating season).

Later on, Li and Lam (5) developed an approach for calculating solar heat gain factors for the horizontal and vertical surfaces based on solar angles and clearness index for sizing air-conditioning equipment. The Hong Kong data were used and a whole year and a cooling season correlations were proposed. Hijazin (6) proposed a correlation that enables a better evaluation procedure in solar design for Amman, Jordan based on local data measurements. These studies and others identified by Jacovides et al (7) developed a specific model taking into account the local climatic differences. On the other hand, based on a four year Toronto's (Canada) data base of the global and diffuse solar radiation on a horizontal surface Orgill and Hollands (3) proposed a correlation model that is suitable for latitude values between 43°N and 54°N. Jacovides et al (7) identified also two more studies where the models were based on a wider geographical area. Several other correlations can be found in the technical reference literature.

This paper evaluates the accuracy of 14 different (kt, kd) correlation models based on the clearness index concept, used as a tool for the estimation of solar irradiation on a vertical south-facing surface and compares the estimated values to in-situ measured values. A set of experimental measurements of solar radiation reaching a South oriented vertical surface was performed in a test cell located about 13 km away from

the weather station. The reliability of the estimation is thus evaluated by comparison with the directly measured values. The distance between the experimental installation and the weather station is within the range of 20 km recommended for a more accurate estimation of the daily irradiation as concluded by Rawlin (quoted in Muneer (8)).

#### The Data Analysis

All the hourly values of the global solar irradiation on horizontal surface were collected at Covilhã's meteorological station, located near the local aerodrome (latitude of 40° 16' N, longitude of 7° 29' W and altitude of 483 m above the sea level). The daily global irradiation on horizontal surface, during the observation period, presented a minimum value of 1434 Wh/m2 and a maximum value of 3512 Wh/m2. The sets of hourly data were used to determine the hourly clearness index (kt), which is given by:

$$kt = \frac{I_h}{I_c} \tag{1}$$

where, Ih represents the hourly global solar irradiation on a horizontal surface (Wh/m2) and Io the extraterrestrial hourly solar irradiation on a horizontal surface (Wh/m2). Using the collected data of hourly solar irradiation and the expression (A4), in the Appendix, for the calculation of the extraterrestrial hourly irradiation (or irradiance), the hourly clearness index (kt) was determined for the observation period. The hourly clearness index reached a maximum value of about 0.85. The kt presented a great concentration in the range between 0.70 and 0.80 for more than 50% of the occurrences. The intermediate sky conditions occurred most of the time. An overcast day corresponds to a kt < 0.35 and represents about 9% of the occurrences. A less than 0.2 kt value occurred just 3% of the times. A clear day, with kt > 0.8, occurred 8% of the times, while kt > 0.75 represents about 38% of the occurrences.

A pyranometer with a measurement range from 0 to 2000 W/m2 and an accuracy of <10% was used to measure total solar radiation. It was mounted rigidly on a South-oriented façade (Latitude 40° 16' N; Longitude 7° 29' W; Altitude 464 m above the sea level) located about 13 km away from the weather station. There are no weather stations in every corner and buildings could be hundreds of kilometres away from the nearest one. Estimating incident solar radiation onto these buildings based on weather data may be very changeling. Although, the test cell used in this study is within the range proposed by Rawlin. The measurements were performed from November 2008 to January 2009, using a programmable data acquisition systems Mikromec multisens to store the acquisition data. Only the hours for which a complete set of data was available were considered. Due to sunrise and sunset the first local hour of daylight and the last hour may correspond to an incomplete set of hourly data. Due to the distance between the weather station and the place where the solar radiation on a

vertical surface was measured (about 13 km), sometimes the sky conditions were different from one place to the other. Since it was impossible to monitor the sky conditions in both places, when the difference between the measured value for a specific hour and the estimated one for that hour was higher than 50%, that hour value was removed from the set of data to be treated. This was just an occasional occurrence. Even though some of the regression models are more accurate for monthly data than daily or hourly data, the availability of a solar radiation model in a particular region is very useful in estimating the amount of available solar radiation to predict the operation of different devices that rely on Sun (9).

## The (kt, kd) Correlation Models

As mentioned before, in order to determine the global solar irradiation on a nonhorizontal surface, one must know the values of the two different components of the solar radiation on a horizontal surface, the direct and the diffuse contributions. It was also mentioned that the most common available figure from the meteorological stations is the global solar irradiance value on a horizontal surface, making it impossible to directly estimate the radiation on non-horizontal surfaces. In these circumstances, it is necessary to make use of an indirect methodology such as the one based on the clearness index concept. Several studies showed that it is possible to estimate the amount of each of the two components of solar radiation, in a statistical form, as it was concluded by Liu and Jordan, in 1960, as quoted in González and Calbó (10), and, therefore, to calculate the solar irradiation in any non-horizontal surface. The empirical procedure of Liu and Jordan involves a correlation between the percentages of diffuse radiation in relation to global solar radiation or, in other words, the ratio between the diffuse radiation and the global radiation (kd) that reaches the horizontal surface and the clearness index (kt), both already defined above. The higher the clearness index, the higher the percentage of direct radiation that reaches the Earth's surface will be. Lower clearness indexes occur on cloudy days which correspond to lower percentages of direct radiation on Earth's surface. After Liu and Jordan others studied and proposed correlations between kt and kd based upon a specific local weather or a wider region weather. It is not possible nor practical to investigate a model relating kt and kd for each single place on Earth. To find out what differences a specific model can bring to the estimation of solar radiation for a different place, 14 different models are compared in this paper. Correlations between the hourly values of kt and kd are obtained from the algorithm of Erbs et al (2). They proposed the following correlations for the estimation of the hourly diffuse radiation on a horizontal surface based on the hourly global solar radiation on a horizontal surface:

$$kd=1-0.09kt$$
, for  $kt \le 0.22$   
 $kd=0.9511-0.1604kt+4.388kt^2-16.638kt^3+12.336kt^4$ , for  $0.22 \le kt \le 0.8$   
 $kd=0.165$ , for  $kt > 0.8$  (2)

Hawlader, quoted in Jacovides (7), analyzed a tropical location in Singapore and proposed the following correlations:

$$kd = 0.915$$
, for  $kt \le 0.225$   
 $kd = 1.135 - 0.9422kt - 0.3878kt^2$ , for  $0.225 < kt < 0.775$   
 $kd = 0.215$ , for  $kt \ge 0.775$ 

Reindl et al, quoted in Jacovides (7), based on the data of several European and American locations, presented the following correlations:

$$kd=1.02-0.248kt$$
, for  $\le 0.3$   
 $kd=1.45-1.67kt$ , for  $0.3 < kt < 0.78$   
 $kd=0.147$ , for  $kt \ge 0.78$  (4)

Chandraesekaran and Kumar, quoted in Jacovides (7), used the data of Chennai (formerly known as Madras), in India and proposed the following:

$$kd = 1.0086 - 0.178kt, \text{ for } kt \leq 0.24$$

$$kd = 0.9686 + 0.1325kt + 1.4183kt^2 - 10.1862kt^3 + 8.3733kt^4 \qquad , \qquad \text{for}$$

$$0.24 < kt \leq 0.80$$

$$kd = 0.197, \text{ for } kt > 0.80$$

$$(5)$$

Lam and Li (4) developed a correlation model for Hong Kong. The hourly global and diffuse radiation data measured during a four-year period (1991 - 1994) were used to develop the model described by the following equations for three sky conditions:

$$kd=0.977kt$$
, for  $kt < 0.15$   
 $kd=1.237-1.361kt$ , for  $0.15 \le kt \le 0.7$   
 $kd=0.273$ , for  $kt > 0.7$ 

Another correlation was proposed by Orgill and Hollands (3), suitable for latitude values between 43°N and 54°N and can be expressed as follows:

$$kd=1-0.249kt$$
, for  $kt < 0.35$   
 $kd=1.557-1.84kt$ , for  $0.35 \le kt \le 0.75$   
 $kd=0.177$ , for  $kt > 0.75$  (7)

A data base of the global and diffuse solar radiation on a horizontal surface at Toronto (Canada), from September 1967 to August 1971, was used to find kd. For each range of 0.05 of kt, a mean value of kd was calculated, in order to obtain the pairs (kt, kd) representative of each range. Hijazin (6) adopted Orgill and Hollands' method, but

correlating the weighted average values of the kd's with the kt's, which is represented by the following equation:

$$kd = 0.744$$
, for  $kt < 0.1$   
 $kd = 0.842 - 0.977kt$ , for  $0.1 \le kt \le 0.8$   
 $kd = 0.06$ , for  $kt > 0.8$ 

A total of 3477 values for global and diffuse radiation were used from solar radiation data in Amman, Jordan. Miguel et al, quoted in Jacovides (7), used data from several countries in the area of the Baltic Sea, with the following conclusions:

$$kd = 0.995 - 0.081kt$$
, for  $kt \le 0.21$   
 $kd = 0.724 + 2.738kt - 8.32kt^2 + 4.967kt^3$ , for  $0.21 < kt \le 0.76$   
 $kd = 0.18$ , for  $kt > 0.76$ 

Boland et al, quoted in Jacovides (7), based on Victoria's data, presented the following correlation:

$$kd = \frac{1}{1 + e^{7.997(kt - 0.586)}}$$
 (10)

Later on, Li and Lam (5) developed new correlations for Hong Kong, for the whole year, respectively, as follows:

$$kd = 0.976kt$$
, for  $kt \le 0.15$   
 $kd = 0.996 + 0.036kt - 1.589kt^2$ , for  $0.15 < kt \le 0.7$   
 $kd = 0.23$ , for  $kt > 0.7$ 

Oliveira et al, quoted in Jacovides (7), used the data from São Paulo, Brazil and proposed the following correlations:

$$kd=1.0$$
, for  $kt \le 0.17$   
 $kd=0.97+0.8kt-3.0kt^2-3.1kt^3+5.2kt^4$ , for  $0.17 \le kt \le 0.75$   
 $kd=0.17$ , for  $kt \ge 0.75$ 

Karatassou et al, quoted in Jacovides (7), used the data from Athens, with the following result:

$$kd = 0.9995 - 0.05kt - 2.4156kt^2 + 1.4926kt^3$$
, for  $kt \le 0.78$   
  $kd = 0.20$ , for  $kt > 0.78$  (13)

Soares et al, quoted in Jacovides (7), used the same data as the previous Brazilian study (expression (12)), suggesting new correlations:

$$kd = 1.0$$
, for  $kt \le 0.17$   
 $kd = 0.90 + 1.1kt - 4.5kt^2 + 0.01kt^3 + 3.14kt^4$ , for  $0.17 < kt < 0.75$   
 $kd = 0.17$ , for  $kt > 0.75$ 

Jacovides (7), based on a five year period of solar radiation data in Athalassa's (Cyprus), proposed the following correlation, for the whole year:

$$kd = 0.987$$
, for  $kt \le 0.1$   
 $kd = 0.94 + 0.937kt - 5.01kt^2 + 3.32kt^3$ , for  $0.1 \le kt \le 0.8$   
 $kd = 0.177$ , for  $kt > 0.8$ 

#### Assessment of the (kt, kd) Correlation Models

As explained in chapter 2, using the registered values of hourly solar irradiation on horizontal surface and the expression (A4), in the Appendix, for the calculation of the extraterrestrial hourly irradiation (or irradiance), the hourly values of the clearness index (kt) were determined for the whole observation period. Then the described (kt, kd) correlation models were used to calculate the hourly values of kd. Given the calculated hourly values of kd and the registered values of hourly solar irradiation on horizontal surface, the hourly solar irradiation on a South oriented vertical surface was estimated, using the expressions presented in the Appendix, namely (A5) to (A13). An albedo of 0.3 was considered. Figure 1 shows a comparison, in terms of mean daily values of solar irradiation, between experimental measurements (Measured) and the values of irradiation estimated through the use of the different models.

As it can be seen the results are very close to each other, with the exception of correlation (8), predicting the highest value of them all. For the set of hourly values of global solar irradiation (Ig), which were used to determine the mean daily values presented in Figure 1, a statistical analysis was done to determine the mean bias error (MBE) and the root mean square error (RMSE), defined as follows:

$$MBE = \frac{\sum (Estimated(I_g) - Measured(I_g))}{N}$$
(16)

and,

$$RMSE = \sqrt{\frac{\sum (Estimated(I_g) - Measured(I_g))^2}{N}}$$
(17)

where, N is the number of data and Ig is the hourly solar irradiation on the south facing vertical surface. The results of the statistical analysis of the 14 correlation models are presented in Table 1. A MBE > 0 means that the predicted values tend to overestimate the measured values, whereas a MBE < 0 corresponds to an underestimation of the measured ones. The percentage mean bias deviation MB and the percentage root mean square deviation RMS were also calculated by the expressions presented below and are also shown in Table 1.

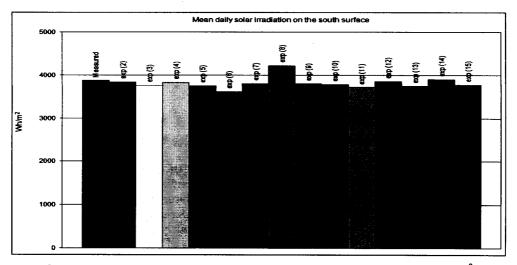


FIG 1. Mean daily solar irradiation on a South oriented vertical surface (Wh/m²) – measured and estimated values.

$$MB = \frac{\sum \frac{\text{Estimated}(I_g) - Measured}(I_g)}{Measured}}{N} 100}$$
(18)

and,

$$RMS = \sqrt{\frac{\sum \left(\frac{Estimated(I_g) - Measured(I_g)}{Measured(I_g)}\right)^2}{N}} 100}$$
(19)

The results show that correlation (8) is the one that deviate the most from the measurements, being correlation (14) the one that approaches the best. The latter correlation leads to a MBE equal to -4 W/m2, a MB of -0.56 %, a RMSE of 48 W/m2 and a RMS of 9.77%. These results indicate an acceptable fitting between the

estimated and the measured values. The differences between the correlations are not very significant. Some deviation from the measured values was of course expected for several reasons. Deviation can occur due to the conditions of the current research that used the data of a weather station that is about 13 km away from the place where the experimental measurements were performed and there were some slight differences in the sky conditions between the two places. Furthermore, as it was explained, the different (kt, kd) correlation models were found for different locations and climates, so some deviation should be expected. However it can be said that the range of error for hourly Ig values that resulted from the use of the 14 (kt, kd) models was quite low. The percentage error in estimated Ig hourly values is calculated as follows:

Expression	MBE Wh/m <sup>2</sup>	RMSE Wh/m <sup>2</sup>	MB %	RMS %
(2)	-16	58	-4.46	12.78
(3)	-29	59	-3.78	13.53
(4)	-17	64	-4.60	12.98
(5)	-30	60	-6.11	12.59
(6)	-51	70	-7.09	12.65
(7)	-22	60	-5.11	12.92
(8)	48	73	13.12	28.59
(9)	-22	59	-5.14	12.57
(10)	-25	68	-5.74	13.82
(11)	-35	60	-5.68	12.34
(12)	-12	52	-2.54	10.29
(13)	-30	61	-4.45	12.14
(14)	-4	48	-0.56	9.77
(15)	-26	59	-4.43	11.42

**TABLE 1.** Results of the statistical analysis of the 14 (kt, kd) correlation models.

$$Error(\%) = \left(\frac{Estimated(I_g) - Measured(I_g)}{Measured(I_g)}\right) \times 100$$
(20)

The average percentage error for the estimated hourly values was of about 13.12% and 0.56% with correlations (8) and (14), respectively. For the whole set of correlations the average percentage error varied from -7.09%, obtained with equation (6) to 13.12%, with equation (8). Figure 2 shows the hourly percentage error obtained by the different (kt, kd) correlation models for three consecutive and complete days. In Figure 2, three specific moments can be observed where the error is higher. Two of them are when the solar radiation intensity is low, at sunrise and at sunset. The third one is in the middle of the afternoon. This is due to the effect of the presence of a lamp post that is located slightly to the west of the test façade providing shadow on the pyranometer resulting in an apparent overestimated value. This is, of course, a circumstantial effect that the models can't predict. Results presented in Table 1 and in

Figure 1 disregarded the values for this specific short period of time. Figure 3 shows another group of three consecutive and complete days where there were some clouds over the weather station.

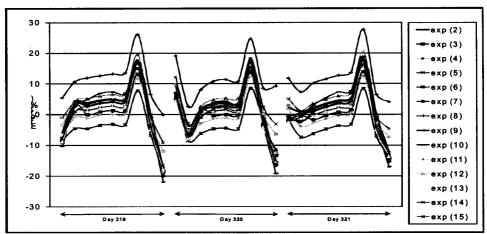


FIG 2. Hourly percentage error of solar irradiation on a South façade (%) from day 319 to day 321.

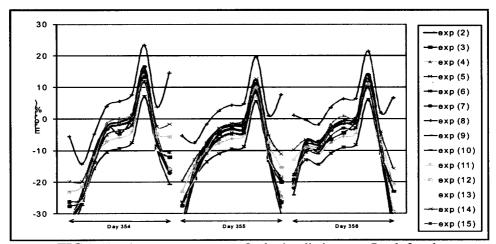


FIG 3. Hourly percentage error of solar irradiation on a South façade (%) from day 354 to day 356.

In Figure 3, the same moments where the error is higher can be observed. In all cases the MB is fairly small, but there are larger errors at hourly intervals, especially when the sky conditions are not equal in both locations. In this analysis the difficulty of

estimating solar radiation is not so much over the full day, but rather when predicting the extreme day events (sunrise/sunset). It seems that the models have less accuracy under low solar altitude which may correspond to a broad range of sky conditions. This also corresponds to a lower solar radiation intensity which leads to an increased magnitude of the percentage error. Doing the exercise of cutting off the first and last complete hours of sunlight, 9 am and 5 pm, the MB presents a range from -5.98% (exp. 6) to 12.00% (exp. 8) while RMS it goes from 9.10% (exp. 14) to 25.98% (exp. 8). RMSE goes from 50 W/m2 (exp. 14) to 80W/m2 (exp. 8), while MBE, considering out those two hours, it goes from -54 W/m2 (exp. 6) to 54 W/m2 (exp. 8).

### Conclusion

The estimation of the available solar energy on non-horizontal surfaces, for a specific location, is possible even when the only available data are the values of the global solar radiation on horizontal surface. Since it is impossible in practice to define a correlation between kt and kd for each place on Earth, a correlation defined for a near place can be chosen. In what concerns the climatic data to be used for a specific location, some caution is needed regarding the distance from the weather station and regarding different sky conditions, especially at sunrise and sunset. Horizon elevation and specific surroundings must be carefully evaluated too. The registered values of global solar radiation on horizontal surface from a weather station and the measured values of the incident solar radiation on a vertical south-oriented surface, 13 km apart from each other, were used to compare the accuracy of 14 (kt, kd) correlation models to predict the solar radiation incident on the vertical south-oriented surface. The accuracy of each individual (kt, kd) correlation model was evaluated through statistical and also graphical analysis, based on hourly data. This kind of approach proved to have an acceptable precision. It was expected that correlation 9, studied for a range of countries as Portugal, would fit better to the measurements, however, the correlation 14, initially studied for Brazil, S. Paulo, southern hemisphere, has shown the best approach. On the contrary the correlation 8 seems to be the one that further away from the measurements. As kt/kd correlations depend on the atmosphere conditions, local pollution may also affect any prediction based on these expressions, nevertheless, is a worldwide accepted method.

#### References

- 1. Manuel Collares-Pereira, Ari Rabl, The average distribution of solar radiation correlations between diffuse and hemispherical and between daily and hourly insolation values. Solar Energy 22 (1979) 155-164.
- 2. D. G. Erbs, S. A. Klein, J. A. Duffie, Estimation of the diffuse radiation fraction for hourly, daily and monthly average global radiation. Solar Energy 28 (1982) 293-302.
- 3. J. F. Orgill, K. G. T. Hollands, Correlation equation for hourly diffuse radiation on a horizontal surface. Solar Energy 19 (1997) 357-359.

- 4. Joseph C. Lam, Danny H. W. Li, Correlation between global solar radiation and its direct and diffuse components. Building and Environment 31 (1996) 527 535.
- 5. Danny H. W. Li, Joseph C. Lam, Analysis of solar heat gain factors using sky clearness index and energy implications. Energy Conversion & Management 42 (2001) 555 571.
- 6. Maher I. Hijazin, The diffuse fraction of hourly solar radiation for Amman / Jordan. Renewable Energy 13 (1998) 249 253.
- C. P. Jacovides, F. S. Tymvios, V. D. Assimakopoulos, N. A. Kaltsounides, Comparative study of various correlations in estimating hourly diffuse fraction of global solar radiation. Renewable Energy 31 (2006) 2492-2504.
- 8. T. Muneer, Solar radiation and daylight models. Elsevier Butterworth-Heinemann, Oxford, 2004.
- Hassan A. N. Hejase and Ali H. Assi, Time-Series Regression Model for Prediction of Mean Daily Global Solar Radiation in Al-Ain, UAE, ISRN Renewable Energy Volume 2012, Article ID 412471, p. 11.
- 10. Josep-Abel González, Josep Calbó, Influence of the global radiation variability on the hourly diffuse fraction correlations. Solar Energy 65 (1999) 119-131.
- 11. Fernando Branco, Pedro Mendes, Térmica dos edificios. Instituto de Soldadura e Qualidade, Oeiras, 1996.
- 12. Manuel Collares Pereira, Hélder Gonçalves, Pedro Paes, Método de cálculo para dimensionamento de sistemas destinados ao aproveitamento de energia solar para o aquecimento de água por intermédio de colectores planos. Laboratório Nacional de Engenharia e Tecnologia Industrial, Amadora, 1983.
- 13. John A. Duffie, William A. Beckman, Solar engineering of thermal processes. John Wiley & Sons, Inc., New Jersey, 2006.
- 14. Greg Stickler, Solar Radiation and the Earth System. NASA National Aeronautics and Space Administration (2007) www.nasa.gov/ (accessed in November, 2007).
- 15. Cláudio Monteiro, Integração de Energias Renováveis na Produção Descentralizada de Electricidade Utilizando SIG. MSc dissertation, Faculty of Engineering of the University of Porto, 1996.
- Manuel Collares Pereira, Radiação Solar. Centro para a Conservação de Energia, Amadora, (no date).
- 17. World Radiation Centre, Solar constant (2006) http://www.pmodwrc.ch/pmod.php?topic=tsi/composite/SolarConstant (accessed in November, 2007).
- 18. Álvaro Silva Ribeiro, Instrumentação de uma estação de medição de iluminação natural. Laboratório Nacional de Engenharia Civil, Lisboa, 2000.
- ASHRAE American Society of Heating, Refrigeration and Air-Conditioning Engineers, Inc., ASHRAE handbook - 1985 fundamentals. American Society of Heating, Refrigeration and Air-Conditioning Engineers, Inc., Atlanta:, 1985.
- 20. University of Oregon, Solar Radiation Monitoring (2002). http://solardat.uoregon.edu/. (accessed in December, 2004).