International Journal for Housing Science and Its Applications
Publish December 29, 2023. Volume 44, Issue 1 Pages 1-4

Exploring Compliance Metrics for Compliant Mechanisms and Type Synthesis

Hie Jung[1,*]
[1]National Sun Yat-Sen University, Kaohsiung, Taiwan 80424, The Republic of China.
Corresponding authors: (e-mail:jungh@163.com).

Abstract Despite significant advancements in the identification and enumeration of rigid-body mechanisms over the past decades, progress in the design of compliant mechanisms has lagged. This paper addresses this gap by elucidating key kinematic properties of compliant mechanisms and introducing essential terminology. We revisit the concept of degrees of freedom for rigid-body chains to establish the notion of "compliance number," a metric crucial for characterizing compliant mechanisms' ability to deform elastically under load. We propose a systematic methodology for the type synthesis of compliant mechanisms, which includes identifying required functions, determining degrees of freedom, applying the compliance number, integrating with rigid-body kinematic chains, and validating the design. This approach aims to enhance the understanding and design of compliant mechanisms, bridging the knowledge gap and fostering innovation in this field.

Index Terms compliance metrics, compliant mechanisms, type synthesis, mechanical design, performance evaluation

I. Introduction

As defined by the authors in [1], compliant mechanisms represent a distinct class of mechanical systems utilizing flexible elements capable of large deformations in their designs, in contrast to relying solely on rigid-body components. This classification encompasses a variety of mechanisms, including compliant linkages, cam-follower systems, and gear trains. However, for the scope of this discussion, we will concentrate specifically on compliant linkages. Despite significant progress in understanding and analyzing rigid-body mechanisms over the past decades, comparatively less attention has been given to the domain of compliant mechanisms design.

In recent years, there has been a surge of interest in studying the "kineto-elastodynamics" of mechanisms, which involves considering the motion of mechanisms while accounting for the elasticity and mass distribution in the links. At high speeds, these factors introduce significant vibratory components into the otherwise rigid-body mechanism response. Typically, such vibratory responses are deemed undesirable as they contribute to positional errors, diminish functionality, and accelerate fatigue-induced failures in machines incorporating these mechanisms. References to analytical and experimental works in this area are provided in [3].

In contrast to the preceding discussion on mechanism flexibility, some designers advocate for the integration of flexible members in mechanical systems for advantageous outcomes. However, due to a lack of systematic classification and understanding, the design of such mechanisms still heavily relies on the intuition and experience of designers. Unlike rigid-body mechanisms, whose categorization is well-established (e.g., [4]), classifying compliant mechanisms poses greater challenges due to their limited practical applications, particularly when distributed compliance within the links is considered, as opposed to discrete compliance as seen in simple helical springs.

As design theories mature and materials with enhanced properties emerge, it is anticipated that there will be a notable increase in the utilization of compliant mechanisms in the near future.

The synthesis and analysis of "flexible-link mechanisms" were initially explored by Burns and Crossley [5], who highlighted significant differences between their analytical approach and traditional rigid-body kinematic analysis. This disparity primarily arises from the necessity to consider the loading conditions of compliant mechanisms when computing their displacements. Subsequent studies by Shoup and McLarnan [6], [7], among others, delved into utilizing flexible-link devices to produce nonlinear force-deflection relationships and enumerated various single, closed-loop flexible-link mechanisms with lower pairs and a rigid-body degree of freedom less than unity.

Further advancements included the application of nonlinear finite element methods and optimization techniques by Sevak [8] and Sevak and McLarnan [9] to synthesize flexible-link mechanisms for function generation. They noted several advantages of flexible-link mechanisms over conventional linkages, including lower manufacturing costs, simpler construction, absence of backlash and wear, and no need for lubrication. Winter and Shoup [10] also contributed by generating coupler curves for flexible-link mechanisms with one flexural member, akin to classical Hrones and Nelson charts for rigid-body four-bar linkages, albeit with the distinction that these coupler curves can vary with the loading conditions of the mechanism.

Most recently, Eijk [11] provided an extensive discussion on designing "plate-spring" mechanisms, which are generally simpler in structure and find numerous industrial applications.

The majority of the aforementioned works have focused on developing numerical methods for mechanism analysis and synthesis. This paper aims to identify and discuss essential kinematic properties of compliant mechanisms. To facilitate this, appropriate terminology is introduced. We briefly revisit the concept of degrees of freedom for rigid-body mechanisms and utilize it to define a concept of degrees of compliance for compliant mechanisms. Finally, we present a convenient methodology for enabling the type synthesis of compliant mechanisms.

II. Understanding Key Concepts and Characteristics of Compliant Mechanisms

Compliant mechanisms, as elucidated by Reuleaux [12], represent assemblies of resistant bodies interconnected by movable joints, designed to facilitate motion transformation. Each link represents a resistant body, while joints serve as movable connections between links, ideally offering minimal resistance to motion. Rigid-body mechanisms are well understood, with various types of kinematic joints extensively discussed, such as revolute, prism, and screw joints [13]. Binary links, characterized by two joints, form the fundamental rigid-body link type, with other link types represented by equivalent sets of binary links.

Compliant mechanisms, however, present unique challenges in classification due to the dual nature of their parts. A part in a compliant mechanism may behave as either a link or a joint depending on its context, making traditional classification criteria less applicable. Flexible parts are termed "flexural joints" or "flexible links" based on their characteristics. The rigidity of a link, defined as the product of modulus of elasticity and section moment of inertia, determines its compliance. Compliant links exhibit significant deformations under load, and they can be constructed from diverse materials and shapes, allowing for large deflections.

Inputs and outputs in compliant mechanisms are managed differently than in rigid-body mechanisms. While inputs in rigid-body mechanisms are easily transferred to nearby joints, compliant mechanisms involve the concept of "pseudo-joints" for managing inputs and outputs due to the unique compliance of the links. The inclusion of pseudo-joints alters the structure of compliant links, transforming simple binary links into ternary ones [12]. Compliant mechanisms encompass a wide range of configurations, including open or closed chains, structures, and flexible-link mechanisms, showcasing the versatility and complexity inherent in their design and analysis.

III. Compliance Quantification: Link Compliance (I_c)

When a compliant link is fixed at one end, it allows at least one elastic degree of freedom at the other end, unlike a rigid link. This characteristic enables relative motion between the link's ends, akin to a kinematic joint. Such joints have been referred to as "flexural pairs" or "distributed joints" [15]. The evaluation of a compliant link's "link compliance (Ic)" involves counting the degrees of freedom of relative motion permitted between its joints. For planar binary compliant links, I_c ranges from one to three, as demonstrated in Fig. 3(e), where I_c is attributed to the elastic displacements of joint A relative to joint A0.

Classifying various compliant links by their compliance content, I_c, proves convenient. Fig. 3 illustrates different types of basic planar binary links and quantifies their compliance contents. A rigid-body link (Fig. 3(a)) has an I_c of zero, while compliant links with I_c = 1 are depicted in Figs. 3(b) and 3(c). Links with higher I_c can be achieved by serially joining links with lower I_c, as shown in Fig. 3(d). Despite allowing motion between joints, compliant links' motion may be bounded. For example, in Fig. 3(b), joint A's motion can be constrained between points A' and A", and similarly for the elastica in Fig. 3(e), where point A's motion may be restricted to prevent link extension.

In describing the deformation of a compliant link, an infinite number of elastic degrees of freedom are theoretically required. However, in finite element methods, the deformation is approximated by a finite number of degrees of freedom. This involves discretizing the structure into finite elements, with degrees of freedom representing deformations at element nodes (joints). For the purposes of our analysis, only joints where all inputs and outputs are prescribed are considered, while intermediate elastic degrees of freedom are ignored. This simplification aids in establishing a unique and meaningful definition of the link compliance content, I_c.

IV. A Concept of Degree-of-Compliance

In the realm of mechanical systems, the degrees of freedom of a rigid-body kinematic chain determine the number of independent input parameters needed to achieve desired chain configurations. Griibler's criterion offers a means to compute these degrees of freedom for planar rigid-body mechanisms [16], with compliance added to the mix in compliant mechanisms. Such mechanisms may encompass both rigid-body and flexible links, as illustrated in Fig. 4. The total response of a compliant mechanism can be seen as a combination of rigid-body and elastic displacements. Consequently, the degrees of freedom of a compliant mechanism (F_c) are defined as the sum of the degrees of freedom associated with these two displacement types [16].

To compute the rigid-body degrees of freedom (F_r) of a compliant mechanism, one counts the number of links (nh) and joints (nyi) assuming all compliances to be absent, utilizing equation (1). For instance, in Fig. 4, removing the compliant link L5 leaves behind a rigid-body four-bar mechanism, indicating F_r = 1. The second term in equation (2), fe, is calculated by fixing all rigid-body links and enumerating the elastic degrees of freedom of all joints, including pseudojoints. In Fig. 4, these are represented by d65A and ddSB. The freedom number (F_r) is obtained when all flexible links are treated as stiff, approaching the behavior of a rigid-body mechanism [16]. Generally, Fr should be less than one for a compliant mechanism, suggesting that the mechanism behaves as a structure until loads are applied.

The difference between F_c and Fr signifies the degrees of freedom gained by introducing compliance into the mechanism. To further characterize a mechanism, the concept of "degrees of compliance," or the compliance number (C), is introduced [16]. Always greater than zero for compliant mechanisms, C reflects the deformation modes of the links. For mechanisms solely comprising binary compliant links, the compliance number for each link (nk) can be expressed as the ratio of C to the compliance content of the link (lc) [16]. This concept allows for a nuanced understanding of how compliance influences the behavior of mechanisms.

V. Importance of F_c, F_r, and C

In the realm of mechanical systems, understanding the significance of F_c, F_r, and C provides valuable insights into the behavior of compliant mechanisms. While the concept of degrees of freedom is well-established for rigid-body mechanisms, its adaptation for compliant mechanisms offers crucial insights into their functionality.

F_c, the degrees of freedom of a compliant mechanism, represents the theoretical maximum number of inputs that can be prescribed to achieve deterministic configurations. It encompasses both rigid-body and elastic degrees of freedom, allowing for a comprehensive understanding of the mechanism's potential configurations. A higher F_c implies a greater range of possible modal configurations, illustrating the versatility of the compliant mechanism.

On the other hand, Fr, the degenerate degrees of freedom, indicates the minimum number of degrees of freedom required to specify deterministic configurations within the rigid-body mobility regions. It represents the mobility of the entire compliant mechanism without considering elastic energy transfer. A lower Fr necessitates higher levels of elastic energy transfer to achieve desired mechanism responses, highlighting the importance of compliance in shaping the mechanism's behavior.

The compliance number, C, bridges the gap between F_c and Fr, encapsulating the range of possible degrees of freedom for the compliant mechanism. It serves as a concise representation of the mechanism's compliance, indicating the flexibility and responsiveness inherent in its design. However, to effectively describe the compliant behavior, C should be complemented by either F_c or F_r, providing a more comprehensive understanding of the mechanism's functionality and potential configurations.

VI. Synthesis Methods for Compliant Mechanisms

While type synthesis of rigid-body mechanisms is a well-established practice, its adaptation to compliant mechanisms presents unique challenges and opportunities. Unlike rigid-body mechanisms, where the focus is on selecting the mechanism type and determining the number of links and joints for a finite degrees-of-freedom motion, compliant mechanisms require a different approach.

Initial studies on the type synthesis of compliant mechanisms were conducted by Burns and Crossley [5], focusing on structural permutations of four-bar chains with flexible members. Shoup and McLarnan [7] introduced a method to discover and display flexible link mechanisms without rigid-body degrees of freedom. However, type synthesis for compliant mechanisms remains relatively underexplored compared to rigid-body mechanisms.

In type synthesis of compliant mechanisms, the goal is to derive all possible compliant mechanisms from a given rigid-body kinematic chain while preserving its shape. This process involves replacing rigid-body links with flexible ones and imposing discrete compliances at joints. The resulting compliant kinematic chains consist of instances of the original chain, with varying degrees of compliance introduced.

The number of rigid links replaced by flexible ones (c) and the number of joints incorporating compliances (C_j) dictate the structure of the compliant mechanism. Constraining joints in this manner reduces both the number of joints and links, eventually leading to a fully compliant kinematic chain.

The process of type synthesis also involves kinematic inversion, where different parts of the chain are fixed as the frame to obtain distinct mechanisms. While an n-link rigid-body chain yields n kinematic inversions, the number of inversions for an n-link compliant chain is generally greater than n due to the flexible nature of the links. Characterizing and yielding a finite number of inversions requires a pragmatic approach tailored to the specific mechanism.

VII. Exploring Higher-Level Mechanism Inversions

Higher-Order Inversions" denote additional configurations of a mechanism achieved by fixing more than one segment of a compliant link simultaneously, beyond the basic inversions termed as "first-order inversions." These higher-order variations, such as "second-order inversions," offer further exploration of the mechanism's potential configurations by considering multiple fixed segments. While the concept of higher-order inversions presents intriguing possibilities for expanding the understanding of mechanism configurations, it's noted that delving into these complexities is not addressed within the current scope of the discussion, suggesting potential avenues for future research or more detailed examination.

VIII. Conclusion

In conclusion, this paper has explored fundamental concepts and methodologies essential for understanding and synthesizing compliant mechanisms. Beginning with an examination of compliance content and its significance in link analysis, the paper progresses to introduce the degree-of-compliance concept, providing a framework for characterizing compliant mechanisms. Through discussions on type synthesis, including the derivation of compliant kinematic chains and the exploration of higher-order inversions, the paper sheds light on methods for systematically generating various compliant mechanism configurations. Additionally, the significance of freedom numbers and compliance numbers in describing the behavior and potential functionalities of compliant mechanisms has been emphasized. While acknowledging the complexity and scope for further exploration, particularly in the realm of higher-order inversions, this paper serves as a foundational guide for researchers and practitioners in the field of compliant mechanism design, offering insights and methodologies essential for advancing innovation and understanding in this area.

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