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A graph theory-based financing decision path optimization model in a new supply chain finance environment

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Abstract This paper points out the main players in the supply chain finance model, uses Dijkstra's algorithm to calculate the shortest path between each node in the initial supply chain network, incorporates the META graph theory method, constructs the supply chain META graph, and calculates the core enterprise based on the matrix of the supply chain META pathway. Consider the impact of default risk on the optimal financing decisions of distributors, manufacturers and banks under the supply chain financing model of inventory pledge. Propose the optimization path of enterprise financing decision scheme under the collateral credit supply chain financing model. Combined with the numerical simulation method, the evolution path of the three parties is analyzed to explore the influence of different variable values on the financing decision. Under the order pledge financing model, the dealer's optimal order quantity is inversely related to the bank loan interest rate. The higher the loan interest rate is, the higher the financing cost of the dealer is, leading to a decrease in its order quantity. When $R_3 > R_2 + Ar^2$, the probability of the enterprise's choice of loan, the probability of the supplier's choice of fulfillment both tends to 1. The larger the penalty, the faster the tendency to 1. That is, both parties will choose a cooperative strategy (loan, performance). Since suppliers embedded in supply chain finance, the cost of default is increased, and they choose to perform in the long term, so supplier credit will form a benign development in the supply chain finance financing model.

Index Terms meta graph theory, default risk, inventory pledge, financing decision, supply chain finance

I. Introduction

As an innovative financial service model, supply chain finance plays an important role in promoting the deep integration of financial capital and supply chain operation [1], [2]. The core of this model lies in embedding financial services into various links of the supply chain, realizing the synergistic operation of capital flow, information flow and logistics by integrating the resources of the upstream and downstream of the industrial chain, so as to improve the efficiency and competitiveness of the whole supply chain [3]-[5]. It not only helps to alleviate the problem of "difficult and expensive financing" for small and medium-sized enterprises, but also helps core enterprises give full play to the advantages of "chain length", stabilize intra-chain cooperation, and assist enterprises to complete the call for "supply-side structural reform" and "digital transformation" from the perspective of the industrial chain, so as to truly become a financial service entity [6]-[9]. The “multi-win” nature of supply chain finance has greatly stimulated the development of its supply and demand market, coupled with technological empowerment and policy guidance, the cooperation barriers between chain participants are accelerating the dissolution of the chain, and the inter-chain competition is becoming increasingly fierce [10], [11]. How to tighten intra-chain cooperation through supply chain finance business, how to make optimal decisions in supply chain finance business, and how to take into account the participants' preferences for the secondary distribution of benefits have become the focus of academic research, enterprise practice and policy regulation.

Some scholars have revealed the role of digital technology in reshaping the ecology of supply chain finance, which provides theoretical support and practical reference for financing decisions in the supply chain finance environment. Tseng, M. L. et al. explored the financing model under the condition of combining the concept of sustainable development and supply chain finance, and used fuzzy approximation of ideal solution well-preferred ordering technique (fuzzy TOPSIS) to analyze the relevant factors affecting the sustainable supply chain finance model, which provides precise guidance for the enterprise's financing strategy [12]. Wang, Z. et al. constructed a decision support system for supply chain finance based on deep reinforcement learning and particle swarm optimization (PSO) algorithms, which is able to quickly capture the dynamic data of key nodes of the supply chain for analysis and evaluation, and then accurately identify the risks and opportunities in the complex financial environment, providing inspiration for related financing decisions [13]. Yan, N. et al. designed an optimal centrality strategy for supply chain finance to build a multi-attribute decision support model with the objective of maximizing the expected profit and service level and minimizing the bankruptcy cost in order to generate the optimal financing scheme [14]. Sang, B. examined the optimization method of enterprise financing decision from the perspective of credit risk assessment, using genetic algorithm, support vector machine and BP neural network to assess the credit risk of supply chain finance for SMEs, and then proposed the financing decision with high efficiency and benefit [15]. Liang, X. et al. improved the traditional assessment of supply chain financing with economic benefits as the indicator, and established an evaluation model of SME financing under the perspective of sustainable development from three aspects: economic, environmental and social, which not only maximized the economic benefits of financing, but also promoted the sustainable development of SMEs [16]. Lei, Y. et al. proposed a supply chain finance risk identification model based on machine learning algorithms to help decision makers formulate the best financing decisions applicable to the current supply chain operations and development, which not only improves the overall operational efficiency of the supply chain, but also effectively reduces the risk of corporate bankruptcy [17]. It can be seen that the digital technology mentioned in the above study not only changes the business process of supply chain finance and improves the efficiency and accuracy of enterprise financing services, but also profoundly affects the business ecology and operation mode of supply chain finance.

This paper analyzes the supply chain amount model as well as the supply chain financing model, and considers the impact of default risk on the optimal decision of financing among the three parties: dealers, producers and banks under the inventory pledge model. It proposes the three main participants in the inventory pledge supply chain finance model, considers the producer as the core enterprise, assumes that the distributor, the producer and the bank all have risk-neutral preferences, information symmetry in the supply chain, and make decisions with the goal of maximizing their own profits, and solves the optimal strategies of the three parties by using the inverse induction method. The META graph theory is introduced, and Dijkstra's algorithm is used to calculate the shortest path between each node in the supply chain network, and the aggregation degree of each node is obtained and then the core enterprises in the supply chain are calculated. Analyze the system evolution path of financing tripartite and the influence of different variables on the evolution results.

II. Supply chain finance environment

II. A. Overview of supply chain finance

“Supply chain” is a specialized term that refers to a network of finished and intermediate goods that are produced using raw materials and processed efficiently, and then delivered to customers or to certain enterprises through some kind of sales. Therefore, it can also be said that the supply chain is actually a complete chain of production, purchase and sale as well as transportation. Supply chain finance refers to the provision of corresponding financial services to the supply chain. The overall resources of the supply chain will be effectively integrated with the suppliers, manufacturers and distributors to form a complete set of network chain, to take a scientific and reasonable integration, these efficient synergies to reduce the economic operating costs, improve economic efficiency, and achieve the goal of the supply chain financial ecosystem [18], [19].

In the process of managing the supply chain, due to the supply chain contains a variety of enterprises, the production of a wide range of goods, the number of transactions is high, resulting in a massive amount of transaction data and information, and gradually it will be found in the management of the supply chain process of controlling the logistics and transaction information flow and settlement of the capital flow can be from a new perspective to control the risk of financing, and the effect of this new perspective is excellent, which is also the basis for the formation of the concept of supply chain finance. Therefore, the concept of supply chain finance is formed based on supply chain and supply chain management, and it cannot be called supply chain finance if it leaves the supply chain.

In addition, supply chain finance should not be confused with general trade finance behavior, there will be collection, prepayment and inventory management in the supply chain, and supply chain finance is also based on this. But on the contrary, account collection as well as prepayment and goods collateralization business cannot be taken as supply chain finance, and the concepts of these two must be clarified. Because supply chain finance and the scenario presented by the risk identification and control perspective are different from the previous financial risk control concept.

II. B. Supply chain finance characteristics

Conventional trade credit usually involves banks and other financial institutions specializing in providing financial support to large enterprises or key enterprises that are ranked high in the industry, have good credit standing and have promising development prospects. This credit model is relatively more complicated, and the speed of capital recovery is also relatively slow. Supply chain financing, on the other hand, relies on actual trading activities, with the credit of the core enterprise as the guarantee, and the financial institutions assess the credit of the entire supply chain and provide loans accordingly.

At the level of cultural institutions, the moral factor can be seen as a variable. It is only under institutional constraints that benefits can be obtained from the system and that social balance can be better achieved. Ethical equilibrium means that in an institutional negotiation the economic power of the parties is in a coherent relationship with the transaction as a whole, and that no party generates a profit on the reallocated resources. Therefore, it is necessary to achieve a benign interaction between “emotion” and “reason” to enrich the moral connotation of “new vernacular” in order to achieve “harmony”. The realm of “harmony” can only be achieved. In vernacular societies, the ethical ties between ethnic groups are often condensed into a kind of kinship and nostalgia. In different groups, once different conflicts of interest arise, it is very easy to trigger moral anxiety, i.e., a quantitative and qualitative imbalance in the allocation of moral resources.

There are differences between traditional trade finance and supply chain finance.

First, the traditional trade finance approach is aimed at a single enterprise. And the development direction of supply chain finance is to provide services for many enterprises in the supply chain, which increases the transparency of information and the opportunity of cooperation between enterprises. It also reduces the risk in the financing process.

Secondly, in terms of guarantee mode, supply chain finance is different from the traditional strict guarantee mode. Instead, it takes the core enterprise as the object of guarantee for SMEs and finances SMEs. This mode can well solve the financing difficulties of SMEs due to poor credit status, and it has strong flexibility.

Thirdly, supply chain finance is an effective financing method that can effectively reduce the high default risk due to information asymmetry. This project is based on actual trade and extends credit evaluation to the whole process, both to allow financial institutions to directly participate in the utilization of funds and to consider the actual needs of enterprises. It also controls the distribution of goods. By collaborating with logistics companies, it is possible to effectively monitor the capital flow of logistics companies and reduce possible risks.

II. C. Supply chain finance models

II. C. 1) Bank enterprise-based models

In the Internet financial environment supply chain financial system based on the model of banking enterprises, compared with the previous banking enterprises can ensure the authenticity of data. However, banks have cooperative relationships with many enterprises, and most large enterprises carry out supply chain financial services based on the financing needs of small enterprises. As in the past, enterprises need to provide corresponding supply chain products in the process of transformation, the core enterprise-led logistics providers, suppliers will provide certain financial support. In the banking enterprises as the basis of the leading mode, the Ping An Bank launched the orange e network, for example, the site is a new model relying on the industrial chain, able to provide free information dissemination for small and medium-sized enterprises, the use of auditing and logistical accounting and other institutions for cooperation. Orange e-net currently launches more supply chain finance businesses, such as the Haier Group's procurement free loan platform, which continuously integrates e-commerce and supply chain finance to form a new mode of financing.

II. C. 2) Logistics provider-based models

Logistics providers and banks form a cooperative management between them, using logistics credit guarantee to provide financial services for granting credit, including logistics companies to provide output supervision, in-transit supervision and other elements. There are also some logistics companies that provide guarantees for their customers with cargo control. The essence of the model based on logistics providers is that banks utilize logistics companies to control financial risks, and banks expand their customer base and increase their business space through cooperation with third-party logistics companies.

II. C. 3) Multi-party synergistic development model

Multi-party collaborative development model mainly includes dealers, suppliers, through the core enterprise's own creditworthiness to attract loans, dealers and suppliers under the name of the core enterprise to provide credit financial services, is the most commonly used in the supply chain finance and the typical financing model.

At present, this mode is mainly applied in the automobile and steel industries with relatively perfect supply chains, which are closely related to the core enterprises and have developed corresponding systems. However, when applying this mode, there must be a core enterprise, and the awareness of supply chain management must be sufficient to fully cooperate with the bank credit process.

III. Decision-making on order pledge financing for supply chain finance

III. A. How the inventory collateralization model works

The supply chain financing model, inventory pledge model and credit guarantee model can be summarized by the melt-through warehouse. Specifically speaking, the operation principle of the financing warehouse is that the financing enterprise first deposits the products it owns (raw materials, finished products, purchased goods, etc.) as collateral into the financing warehouse opened by the third-party logistics, and then obtains the bank's credit based on it, and then repays the loan in phases in the process of selling the commodities. The third party logistics company plays an important role. It not only provides general logistics services for dealers, but also provides agency services for banks. For example, it is responsible for the custody of the pledge, value assessment and supervision of the destination of the pledge.

Specifically, the financing warehouse can be further subdivided into two basic modes of operation. One is pledge secured financing and the other is credit secured financing. Next, the operation process of pledge guarantee financing is briefly introduced: here mainly involves three participants: banks, third-party logistics enterprises and financing customers. The provider of financing is the bank, the demand side of financing is the customer, and the customer is also the provider of the pledge. The third party logistics enterprise is the intermediary of the financing warehouse.

The schematic diagram of inventory collateralized supply chain finance model is shown in Figure 1. The specific operation process is: first of all, the financier (customer) will put the goods for sale or inventory into the warehouse to form a pledge, so as to apply for a loan to the bank. At the same time, the third-party logistics enterprise is responsible for acceptance of the goods, value assessment and accordingly to the bank to provide proof of the pledge of movable assets. The bank then issues the loan based on the loan application and the movable property pledge certificate. The financier has to use or sell the goods in the financing warehouse under the supervision of the third party logistics. Specifically, the third party logistics can only allow the financier to pick up the goods if it ensures that the financier will return the sales funds to the account set by the creditors. In the event of default by the financier, the creditors have the right to prioritize the disposal of the pledged goods.

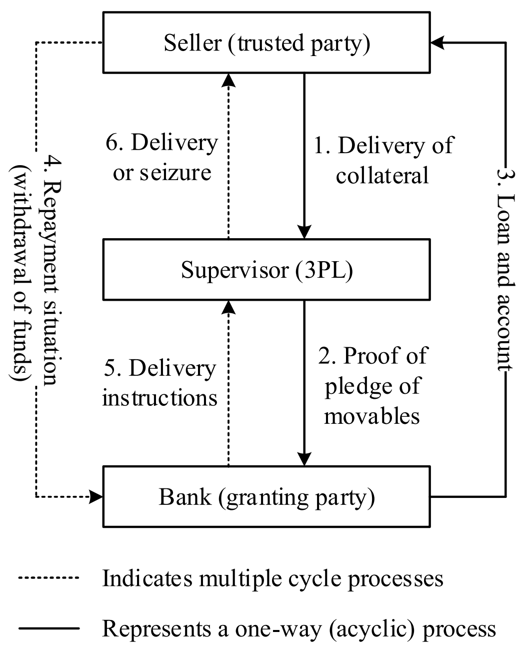


Figure 1: Inventory mortgage supply chain financial model schematic

III. B. Order Pledge Financing Model with Default Risk Consideration

III. B. 1) Relevant assumptions

In a supply chain with a producer as the core firm, it is assumed that the distributor, producer, and bank all have risk-neutral preferences, and information in the supply chain is symmetric, and they make decisions with the goal of maximizing their own profits. The producer and the bank first decide the repurchase rate and the lending rate, and the distributor decides the order quantity accordingly, and the backward induction method is used to solve the optimal strategy of the three parties. Due to the uncertainty of market demand, the dealer has the possibility of default, which increases the bank's loan risk. In order to enhance the dealer's credit, the manufacturer buys back a certain percentage of the dealer's unsold products at the end of the period at the wholesale price.

To simplify the analysis, the value of the remaining product is zero. If the distributor's sales revenue and the repurchase payment made by the producer are insufficient to repay the bank's borrowing, the bank faces two risky scenarios:

(1) The distributor does not go bankrupt and covers the loss on this pledge financing with funds from the company's other operations.

(2) The dealer goes bankrupt and defaults, and the bank will receive all of the dealer's sales proceeds and the repurchase payments made by the manufacturer, and will bear the loss of the dealer's unpaid value.

To ensure the existence of an optimal solution to the model, the demand distribution function $F(x)$ is assumed to be continuously derivable and strictly increasing, and the demand probability density function $f(x)$. $F(x)$ conforms to a distribution with the increasing failure rate (IFR) property, i.e., $G(x)=\frac{xf(x)}{F(x)}$ is monotonically increasing with x, where $F(x)=1-\overline{F}(x)$, in order to ensure that the objective function solution exists and remains unique. To make the trade work, let $P>w(1+r)>c$.

III. B. 2) Dealer Optimization Decision

After a distributor has financed a product order with an order pledge, the ability to repay the bank loan at the end of the sales period depends mainly on the level of demand in the product market due to the uncertainty of future market demand. There is a critical value of market demand z, when the market demand is greater than z, the dealer can repay the loan. When market demand is less than z, the dealer's ability to repay the loan in full depends on whether it defaults or not.

Find the critical value of market demand according to the following equation:

$$px+\theta w(q-x)=(wq-B)(1+r) \tag{1}$$

where θ is the repurchase ratio.

According to the above equation, the critical value z of market demand is obtained:

$$z=\frac{(wq-B)(1+r)-\theta wq}{p-\theta w} \tag{2}$$

When the market demand is less than z, if the dealer does not default, the company's funds are required to cover the loss incurred on the financing transaction $(wq-B)(1+r)-px-\theta w(q-x)$.

When the market demand is less than z, if the dealer defaults, the dealer pays the bank all the sales proceeds and the repurchase amount $px+\theta w(q-x)$, and no other payments are made after that, and the dealer's profit is zero and the bank bears the corresponding loss.

The dealer's expected profit under decentralized decision-making consists of the following four components:

(1) When market demand > g, the dealer is able to repay the loan with a return of:

$$\begin{aligned}\pi_{br}&=\int_{q}^{\varpi}[pq-(wq-B)(1+r)]f(x)dx\\&=[pq-(wq-B)(1+r)]\int_{q}^{\varpi}f(x)dx\\&=pq-(wq-B)(1+r)-[pq-(wq-B)(1+r)]F(q)\end{aligned} \tag{3}$$

(2) The dealer is able to repay the loan with a return when g > market demand >:

$$\begin{aligned}\pi_{br2}&=\int_{z}^{q}[px+\theta w(q-x)-(wq-B)(1+r)]f(x)dx\\&=\int_{z}^{q}[px+\theta w(q-x)-(wq-B)(1+r)]dF(x)\\&=[px+\theta w(q-x)-(wq-B)(1+r)]F(x)|_{z}^{q}\\&\quad-\int_{z}^{q}dF(x)d[px+\theta w(q-x)-(wq-B)(1+r)]\\&=[px+\theta w(q-x)-(wq-B)(1+r)]F(x)|_{z}^{q}-(p-\theta w)\int_{z}^{q}dF(x)\\&=[pq+\theta w(q-q)-(wq-B)(1+r)]F(q)\\&\quad-[pz+\theta w(q-z)-(wq-B)(1+r)]F(z)-(p-\theta w)\int_{z}^{q}dF(x)\end{aligned} \tag{4}$$

(3) When market demand < z, the dealer does not default and makes up the loss on this pledge financing with funds from the firm's other business, which pays off:

$$\begin{aligned}\pi_{br3} &= -(1-\zeta)\int_0^{\infty}[(wq-B)(1+r)-px-\theta w(q-x)]f(x)dx \\ &= (\zeta-1)\int_0^{z}[(wq-B)(1+r)-px-\theta w(q-x)]d(F(x)) \\ &= (\zeta-1)[(wq-B)(1+r)-px-\theta w(q-x)]F(x)|_0^z \\ &\quad -\int_0^z F(x)d[(wq-B)(1+r)-px-\theta w(q-x)] \\ &= (\zeta-1)[(wq-B)(1+r)-px-\theta w(q-x)]F(x)|_0^z \\ &\quad -(\theta w-p)\int_0^z F(x)d(x)\end{aligned} \tag{5}$$

(4) When market demand < z, the dealer defaults on his return:

$$\pi_{br4}=0 \tag{6}$$

Then consider the dealer's expected profit under default risk:

$$\begin{aligned}\pi_{fbr} &= \pi_{br1}+\pi_{br2}+\pi_{br3}+\pi_{br4} \\ &= pq-(wq-B)(1+r)-[pq-(wq-B)(1+r)]F(q) \\ &\quad +[pq+\theta w(q-q)-(wq-B)(1+r)]F(q) \\ &\quad -[pz+\theta w(q-z)-(wq-B)(1+r)]F(z)-(p-\theta w)\int_z^q dF(x) \\ &\quad +(\zeta-1)[(wq-B)(1+r)-pz-\theta w(q-z)]F(z) \\ &\quad -(\zeta-1)(\theta w-p)\int_0^z F(x)d(x)\end{aligned} \tag{7}$$

Further organizing there:

$$\pi_{fbr}=-(p-\theta w)\int_0^q F(x)dx+\zeta(p-\theta w)\int_0^z F(x)dx+[pq-(wq-B)(1+r)] \tag{8}$$

It follows from the first-order derivative of the distributor's profit π_{br} with respect to the order quantity q :

$$\begin{aligned}\frac{\partial \pi_{fbr}}{\partial q} &= -(p-\theta w)F(q)+\zeta(p-\theta w)F(z)\frac{w(1+r)-\theta w}{p-\theta w}+p-w(1+r) \\ &= -(p-\theta w)F(q)+\zeta w(1+r-\theta)F(z)+p-w(1+r)\end{aligned} \tag{9}$$

This is obtained from the second-order derivative of the distributor's profit π_{br} with respect to the order quantity q :

$$\begin{aligned}\frac{\partial^2 \pi_{fbr}}{\partial q^2} &= -(p-\theta w)f(q)+\zeta\frac{[w(1+r)-\theta]^2}{p-\theta w}f(z) \\ &= (p-\theta w)\zeta\left[\frac{w(1+r)-\theta}{(p-\theta w)}\right]^2 f(z)-f(q)]<0\end{aligned} \tag{10}$$

Let $\frac{\partial \pi_{fbr}}{\partial q}=0$ to obtain the order quantity that maximizes the dealer's profit:

$$q_{br}^{*}=F^{-1}\left[\zeta\frac{w(1+r)-\theta}{p-\theta w}F(z)+\frac{p-w(1+r)}{p-\theta w}\right] \tag{11}$$

III. B. 3) Producer's optimal decision

If the market demand is greater than the order quantity q , the producer does not need to buy back and his profit is $q(w-c)$. If the market demand is less than the order quantity q , the producer buys back a certain percentage θ of the unsold surplus product $(q-x)$ at wholesale price. For computational simplicity, this paper considers the producer's expected profit under default risk, assuming and the value of the surplus product is zero:

$$\begin{aligned}\pi_{fbs} &= q(w-c)-\theta w\int_0^q (q-x)f(x)dx \\ &= q(w-c)-\theta w\int_0^q (q-x)dF(x) \\ &= q(w-c)-\theta w(q-x)F(x)|_0^q - \int_0^q F(x)d(q-x) \\ &= q(w-c)-\theta w\int_0^q F(x)dx\end{aligned} \tag{12}$$

It follows from the first-order derivative of the producer's profit π_{fbs} on the repurchase rate θ:

$$\begin{aligned}\frac{d\pi_{fbs}}{d\theta} &= \frac{\partial \pi_{fbs}}{\partial q}\frac{\partial q}{\partial \theta} \\ &= (w-c)\frac{\partial q_{br}^*}{\partial \theta} - w\int_0^q F(x)dx - \theta w F(q)\frac{\partial q_{br}^*}{\partial \theta} \\ &= [w-c-\theta w F(q)]\frac{\partial q_{br}^*}{\partial \theta} - w\int_0^q F(x)dx\end{aligned} \tag{13}$$

In the producer-centered supply chain decision-making, the producer first determines the repurchase rate, and the distributor chooses the order quantity based on the producer's decision, and here the backward induction method is used to solve for the repurchase rate. According to the dealer's optimal order quantity q_{br}^*, there are:

$$F(q_{br}^*) = \zeta\frac{w(1+r)-\theta}{p-\theta w}F(z)+\frac{p-w(1+r)}{p-\theta w} \tag{14}$$

$$\frac{\partial F(q_{br}^*)}{\partial q_{br}^*}\frac{\partial q_{br}^*}{\partial \theta} = \frac{\partial\left[\zeta\frac{w(1+r)-\theta}{p-\theta w}F(z)+\frac{p-w(1+r)}{p-\theta w}\right]}{\partial \theta} \tag{15}$$

$$f(q_{br}^*)\frac{\partial q_{br}^*}{\partial \theta} = \frac{\partial\left[\zeta\frac{w(1+r)-\theta}{p-\theta w}F(z)\right]}{\partial \theta}+\frac{\partial\left[\frac{p-w(1+r)}{p-\theta w}\right]}{\partial \theta} \tag{16}$$

Remember the AI:

$$\begin{aligned}A1 &= \frac{\partial\left[\zeta\frac{w(1+r)-\theta}{p-\theta w}F(z)\right]}{\partial \theta} \\ A2 &= \frac{\partial\left[\frac{p-w(1+r)}{p-\theta w}\right]}{\partial \theta}\end{aligned} \tag{17}$$

Solve for AI there:

$$\begin{aligned}A1 &= \frac{\partial\left[\zeta\frac{w(1+r)-\theta}{p-\theta w}F(z)\right]}{\partial \theta} \\ &= \zeta\frac{\partial\left[\frac{w(1+r)-\theta}{p-\theta w}\right]}{\partial \theta}F(z)+\zeta\left[\frac{w(1+r)-\theta}{p-\theta w}\right]\frac{\partial F(z)}{\partial \theta}\end{aligned} \tag{18}$$

Remember:

$$B1=\zeta\frac{\partial\left[\dfrac{w(1+r)-\theta}{p-\theta w}\right]}{\partial\theta}F(z)$$
$$B2=\zeta\left[\frac{w(1+r)-\theta}{p-\theta w}\right]\frac{\partial F(z)}{\partial\theta} \tag{19}$$

Solve for B1 there:

$$\begin{aligned}B1&=\zeta\frac{\partial\left[\dfrac{w(1+r)-\theta}{p-\theta w}\right]}{\partial\theta}F(z)\\&=\zeta F(z)\frac{-w(p-\theta w)-w(1+r-\theta)(-w)}{(p-\theta w)^2}\\&=\zeta F(z)\frac{w^2r+w^2-wp}{(p-\theta w)^2}\end{aligned} \tag{20}$$

Since $z=\dfrac{(wq-B)(1+r)-\theta wq}{p-\theta w}$ and q is also a function of θ, we get:

$$\begin{aligned}B2&=\zeta\left[\frac{w(1+r)-\theta}{p-\theta w}\right]\frac{\partial F(z)}{\partial\theta}=\zeta\left[\frac{w(1+r)-\theta}{p-\theta w}\right]f(z)\frac{\partial z}{\partial\theta}\\&=\zeta\left[\frac{w(1+r)-\theta}{p-\theta w}\right]f(z)\left[\frac{\partial q_{br}^*}{\partial\theta}\frac{w(1+r-\theta)}{p-\theta w}+q\frac{\partial\left[\dfrac{w(1+r-\theta)}{p-\theta w}\right]}{\partial\theta}\right]\\&=\zeta\left[\frac{w(1+r)-\theta}{p-\theta w}\right]f(z)\left[\frac{\partial q_{br}^*}{\partial\theta}\frac{w(1+r-\theta)}{p-\theta w}+q\frac{w^2r+w^2-wp}{(p-\theta w)^2}\right]\end{aligned} \tag{21}$$

Solving A2 has:

$$A2=\frac{\partial\left[\dfrac{p-w(1+r)}{p-\theta w}\right]}{\partial\theta}=\frac{-(p-w(1+r))(-w)}{(p-\theta w)^2}=\frac{w[p-w(1+r)]}{(p-\theta w)^2} \tag{22}$$

Summarizing the above solving has:

$$\begin{aligned}f(q_{br}^*)\frac{\partial q_{br}^*}{\partial\theta}&=A1+A2=B1+B2+A2\\&=\zeta F(z)\frac{w^2r+w^2-wp}{(p-\theta w)^2}\\&+\zeta\left[\frac{w(1+r)-\theta}{p-\theta w}\right]f(z)\left[\frac{\partial q_{br}^*}{\partial\theta}\frac{w(1+r-\theta)}{p-\theta w}+q\frac{w^2r+w^2-wp}{(p-\theta w)^2}\right]\\&+\frac{w[p-w(1+r)]}{(p-\theta w)^2}\end{aligned} \tag{23}$$

Substituting the above equation into the first-order derivative equation of the producer's profit π_{bs} with respect to the repurchase rate θ. Let $\dfrac{d\pi_{bs}}{d\theta}=0$, and refer to the derivation of the producer's optimal repurchase rate in the conforming warehouse financing decision to find the producer's optimal repurchase rate for a given distributor's order quantity q_{br}^* and loan interest rate r:

$$\theta^{å} = \frac{2pA_1\int_0^q F(x)dx + \zeta\left[\frac{w(1+r-\theta)}{p-\theta w} - 1\right]A_3A_4 + A_5F(q_{br}^*)}{-2w\left[A_1\int_0^q F(x)dx - \zeta A_2\left[1-\frac{w(1+r-\theta)}{p-\theta w}\right]F(z)\right]} + \frac{\sqrt{-4\Omega_2\Omega_4\Omega_5\int_0^q F(x)dx + \zeta\left[\frac{w(1+r-\theta)}{p-\theta w} - 1\right]\Omega_2\Omega_4 q + \Omega_5 f(z) - \Omega_5F^2(q_{br}^*)}}{-2w\left[\Omega_1\int_0^q F(x)dx - \zeta\Omega_2\left[1-\frac{w(1+r-\theta)}{p-\theta w}\right]F(z)\right]} \tag{24}$$

Among them:

$$\begin{aligned}
A_1 &= f(q_{br}^*) - \zeta\left[\frac{w(1+r-\theta)}{p-\theta w}\right]^2 f(z) \\
A_2 &= -\left[\frac{w(1+r-\theta)}{p-\theta w}\right]q_{br}^* f(z) + \bar{F}(z) - 1 \\
A_3 &= q_{br}^* f(z) + F(z) - 1 \\
A_4 &= pF(\dot{q}_{br}^*) - c + w \\
A_5 &= p - w(1+r)
\end{aligned} \tag{25}$$

The following analyzes the relationship between dealer orders and producer buyback ratios according to the expression $\frac{\partial q_{br}^*}{\partial \theta}$:

$$\frac{\partial q_{br}^*}{\partial \theta} = \frac{\left[p - w(1+r) + \zeta\left[1-\frac{w(1+r-\theta)}{p-\theta w}\right](p-\theta w)\left[q\frac{w(1+r-\theta)}{p-\theta w}f(z) - \bar{F}(z) + 1\right]\right]}{\frac{w}{(p-\theta w)^2}\left[f(q_{br}^*) - \zeta\left(\frac{w(1+r)-\theta}{p-\theta w}\right)^2 f(z)\right]} \tag{26}$$

Among them:

$$\begin{aligned}
&0 < \left(\frac{w(1+r)-\theta}{p-\theta w}\right) \le 1 \\
&\left[f(q_{br}^*) - \zeta\left(\frac{w(1+r)-\theta}{p-\theta w}\right)^2 f(z)\right] > 0 \\
&\left[q\frac{w(1+r-\theta)}{p-\theta w}f(z) - \bar{F}(z) + 1\right] = \left[q\frac{w(1+r-\theta)}{p-\theta w}f(z) + F(z)\right] > 0 \\
&\left[1-\frac{w(1+r-\theta)}{p-\theta w}\right] = \left[\frac{p-w(1+r)}{p-\theta w}\right] > 0 \\
&\left[p - w(1+r)\right] \ge 0
\end{aligned} \tag{27}$$

So $\frac{\partial q_{br}^*}{\partial \theta} > 0$, which indicates that the order quantity of the dealer is positively related to the repurchase rate of the producer, and the higher the repurchase rate is, the higher the dealer's motivation to order, and the more its order quantity increases. Therefore, in the supply chain finance consisting of three parties: producer, distributor and the bank providing order pledge financing, the high or low repurchase rate given by the producer to the distributor has a direct impact on the distributor's order quantity.

III. B. 4) Optimal bank decision-making

When the retailer keeps the contract, the supplier does not need to pay the bank funds, and the bank receives the principal and interest of the loan. When the retailer is over-contracted, the supplier needs to pay the bank a percentage of the funds, and the bank receives the retailer's over-contracted liquidation and a percentage of the supplier's funds. Then the bank profit function is as follows:

$$\pi_b = \begin{cases} \lambda p q_m (r - r_0), x > h \\ px + v(q_m - x) + \beta[\lambda p q_m (1+r) - px - v(q_m - x)] - \lambda p q_m (1+r_0), x \le h \end{cases} \tag{28}$$

Theorem 1: Given the supplier wholesale price w, the credit guarantee ratio β and the retailer's optimal initial pledge volume q_m, there exists a unique prime rate for the bank and the prime rate is:

$$\lambda^* = \frac{(p-v)F^{-1}\left(\frac{r-r_0}{(1-\beta)(1+r)}\right)}{pq_m(1+r)} + \frac{v}{p(1+r)} \tag{29}$$

Proof: the expectation of the bank's profit function is obtained:

$$\begin{aligned} E(\pi_b) &= \int_h^{+\infty} \lambda p q_m (r-r_0) f(x)dx + \int_0^h [px + v(q_m - x) + \beta[\lambda p q_m (1+r) \\ &\quad - px - v(q_m - x)] - \lambda p q_m (1+r_0)] f(x)dx \\ &= \lambda p q_m (r - r_0) - (p - v - \beta p + \beta v)\int_0^h F(x)dx \end{aligned} \tag{30}$$

The first-order, second-order partial derivatives of the pledge rate are obtained for the bank's expected profit function:

$$\begin{aligned} \frac{dE(\pi_b)}{d\lambda} &= pq_m(r-r_0) - (p - v - \beta p + \beta v)\frac{pq_m(1+r)}{p-v}F(h) \\ &= pq_m(r-r_0) - (1-\beta)pq_m(1+r)F(h) \end{aligned} \tag{31}$$

$$\frac{d^2E(\pi_b)}{d\lambda^2} = -(p-v)(1-\beta)\left[\frac{pq_m(1+r)}{n-v}\right]^2 f(h) < 0 \tag{32}$$

Therefore, there is a prime rate. The prime rate is:

$$\lambda^* = \frac{(p-v)F^{-1}\left(\frac{r-r_0}{(1-\beta)(1+r)}\right)}{pq_m(1+r)} + \frac{v}{p(1+r)} \tag{33}$$

The proof is complete.

From Theorem 1, it can be seen that the bank's pledge rate depends not only on the supplier's guarantee ratio, but also on the retailer's initial pledge volume. The higher the supplier's guarantee ratio, the lower the bank's lending risk. The higher the retailer's initial pledge volume, the higher the bank's lending risk. Therefore, the bank's decision on the pledge ratio takes into account the supplier's guarantee ratio and the retailer's initial pledge volume.

Corollary 1: When all other conditions remain constant, for the inventory pledge model under study, the prime pledge rate increases with the increase of the guarantee ratio and the two are positively correlated.

Proof: from equation (33): when the guarantee ratio β increases, $1-\beta$ decreases, $F^{-1}(x)$ is a monotonically increasing function, therefore, λ increases as β increases.

Corollary 1 shows that the guarantee ratio is one of the key factors affecting the bank's decision on pledge rate. The higher the supplier's guarantee ratio, the higher the bank's pledge rate. This is because the higher the supplier's guarantee ratio, the less risky it is for the bank to lend and the less the bank will lose in case of retailer default.

Corollary 2: When other conditions are constant, for the inventory pledge model under study, the prime pledge rate decreases with the increase of the initial pledge volume, and the two are negatively correlated.

Proof: from equation (33): when the initial pledge volume q_m increases, λ decreases. Therefore, λ decreases as β increases.

Corollary 3: shows that the bank's decision on pledge rate takes into account the amount of inventory pledged. The bank's pledge rate decreases as the amount of inventory pledged increases. Considering that in case of default by the retailer, the bank will only get the pledge, in order to reduce the risk, the pledge rate will be reduced in order to reduce the number of loans granted so that the bank can better avoid the risk.

III. C. Financing decision options for enterprises

Based on the above analysis can be obtained in the collateralized credit supply chain financing model of the enterprise's financing decision-making program. Loan interest rate is an important parameter of the enterprise's financing decision, from the surface of the loan interest rate is high, the enterprise's financing cost is high, so the loan amount will fall accordingly. Loan interest rate is low, the enterprise's financing cost is reduced accordingly, so the loan amount will rise accordingly. However, if further in-depth discussion can be found, the bank's lending rate is not an exogenous variable, but depends on a variety of factors, especially the supply chain related to the operational decisions of the parties involved, so the seller enterprise can be used by the use of related strategies to achieve the optimization of financing solutions.

To reduce the cost of financing and to achieve the optimization of the financing scheme the firms can obtain through the following ways:

First, in the selection of pledged commodities, it is necessary to choose products with relatively stable prices. This is because price stability means that there is less risk of price fluctuations in the pledged goods, and thus V the likelihood of a decline in value is reduced. The loan interest rate r changes in the opposite direction to the disposal price of the pledge V, so the likelihood of the loan interest rate r rising is reduced. Thus, in the negotiation process with the bank, the seller can secure a strong position for himself to reduce the cost of financing.

Secondly, in terms of sales pricing, the price is appropriately lowered. The lending rate r varies in the opposite direction to the selling price P_{agency}. This is because in a supply chain with decentralized decision making banks are concerned that too high a selling price raises market risk, so they pressure firms through interest rate signals. Thus appropriately lowering the selling price is also a means of obtaining favorable financing rates.

Third, strengthen the risk response measures of enterprises to effectively resist market risks. In the previous analysis, the variable $\hat{D}$, which examines market risk, was introduced in order to take into account the possibility of sellers defaulting on their loans, and the risk of sellers defaulting on their loans increases when the market demand is too small, i.e., $D < \hat{D}$. The lending rate r varies in the same direction as the variable of market risk $\hat{D}$. Thus high risk inevitably leads to high levels of financing interest rates. For enterprises, market risk $\hat{D}$ is not exactly an exogenous variable, but a quasi-endogenous variable that can be controlled through internal operations management. By strengthening internal risk management, the market risk faced by enterprises can be reduced, thus providing a basis for obtaining favorable financing rates.

Fourth, adopt a prudent ordering strategy. The higher the order quantity, the higher the financing rate. On the contrary, if the order quantity is appropriately reduced, the financing interest rate will fall accordingly.

Fifth, try to keep the purchase price as low as possible when negotiating with suppliers. It can be seen that the lending rate r and supplier price $P_{provider}$ change in the same direction. Therefore, it is beneficial to suppress the purchase price in the negotiation in order to obtain a lower financing rate.

III. D. Application of META graph theory to supply chain finance

III. D. 1) META graph theory

Meta-diagram is a graphical structure used for system description and analysis. The structure combines the advantages of graphical tools such as directed graphs, undirected graphs, hypergraphs, and graphs with or without graphs. The diagram, i.e., it can be used for intuitive graphical representation of various systems as well as for adequate formal description and analysis, and is widely used in the fields of hierarchical modeling, workflow analysis, decision support systems and management [20].

Meta-graphs and directed graphs in traditional graph theory are both composed of nodes and arcs (or edges), which is what they have in common, where the objects under study are represented by nodes, and the relationships between these objects are represented by arcs or edges. However, Meta-graphs are fundamentally different from directed graphs in that ordered pairs of arcs in Meta-graphs consist of a set of two generating elements, and ordered pairs of arcs in directed graphs consist of two single elements.Meta-graphs can be used to study both the relationships between sets of objects and the relationships between objects.

III. D. 2) Core firm identification based on META graph theory

1) Abstract representation of supply chain network graph and node aggregation degree calculation

(1) In order to calculate the node aggregation degree, this paper first explains the concepts of node neighborhood, network aggregation degree and node aggregation degree. The neighborhood v_i of a node A_i is the set of all the nodes in the supply chain network that are directly connected to the node v_i, i.e., the set of points whose distance a_{ij} to the node v_i is one. That is, $A_i = \{v_j \mid v_j \in V, a_{ij} = 1, j = 1, 2, ..., n\}$. Node v_i contraction means that all nodes

within v_i and its neighborhood A_i are turned into a new node consideration. If node v_i is a core node, the connectivity formed by the contraction of this node will be better and tighter.

(2) The network aggregation degree of supply chain SC can be expressed by the following equation:

$$C(SC) = \frac{1}{nl} \tag{34}$$

where l is the node connectivity $l = \frac{1}{n(n-1)/2}\sum_{i=1}^{n}\sum_{j \neq i}^{n} d_{ij}$. d_{ij} denotes the shortest path length starting at node v_i and ending at v_j. n is the number of nodes and $n \geq 2$. The smaller l is, the smaller n is, and the higher the aggregation of the whole network. The network aggregation degree of the supply chain $SC^{'}$ formed after the contraction of node v_i can then be expressed by the following equation:

$$C(SC^{'}) = \frac{1}{(n-k)l^{'}} \tag{35}$$

where k denotes the degree of contraction of node v_i and:

$$k = |A_i| = \sum_{v_j \in A_i} a_{ij} \tag{36}$$

$l^{'}$ is the contracted node connectivity:

$$l^{'} = \frac{1}{n(n-k-1)/2}\sum_{i=1}^{n-k}\sum_{\substack{j=1 \\ j \neq 1}}^{n-k} d_{ij}^{'} \tag{37}$$

(3) Set the aggregation degree of node v_i:

$$c(v_i) = 1 - \frac{C(SC)}{C(SC)} = 1 - \frac{(n-k)l^{'}}{nl} \tag{38}$$

The larger k is, the smaller the size of the network formed by the contraction of node v_i is, and the higher the aggregation degree of node v_i is. If v_i is a core node, it will be located in the shortest path of many nodes, and the average length of the supply chain network after the node contraction will be smaller, and the node aggregation degree will be larger.

(4) Calculation of node aggregation degree

Based on the introduction of the previous section, the following steps can be used to calculate the node aggregation degree.

Draw a simple network structure diagram of SC, including raw material suppliers, primary product producers, final product producers, distributors, customers and so on.

Calculate the shortest path d_{ij} between each node in the initial supply chain network according to Dijkstra's algorithm [21], [22].

Record the shortest path length matrix $D = \left[d_{ij}\right]$ for the initial network.

Record the shortest path length matrix of each node after contraction $Ak = \left[d_{ij}\right]$.

Calculate the aggregation degree $c(v_i)$ of each node according to the above equations.

2) Construction of core enterprise identification index system

Based on the principles of purposefulness, comprehensiveness, comparability, scientificity and representativeness, the core enterprise identification index system can be established by collating the indicators involved in the supply chain management pathway and related literature.

(1) Enterprise's own development capability

First, production-related capabilities (U1), including enterprise scale, innovation and R&D investment, innovation and R&D promotion capability, and production technology application level. Second, procurement-related capabilities (U2), including the level of centralized procurement, influence on suppliers, supplier concentration. The third is distribution-related capability (U3), including on-time delivery rate, cargo breakage rate, and customer concentration.

(2) Supply chain network impact capacity

First, logistics capability (U4), logistics cost ratio, goods turnaround time, inventory cycle, inventory turnaround rate, order fulfillment cycle, order fulfillment rate. The second is the level of informationization (U5), including the proportion of electronic document management, the level of information platform construction. The third is the level of capital flow (U6), including real net assets, operating profit margin, total asset turnover ratio, current asset turnover ratio, accounts receivable turnover ratio, and the enterprise's credit rating in the bank.

(3) Nodal importance level

Nodal importance (U7) mainly refers to the importance of member enterprises in the supply chain network, and can be considered using the “node aggregation degree” indicator.

3) Core enterprise identification based on META graph theory

(1) Assuming that enterprise V5 is the enterprise with the largest degree of aggregation, i.e., the core enterprise of the initial selection, and the supply chain where it is located includes V1, ..., V9, and the META of the supply chain is shown in Figure 2. The eligible META pathways in the figure are META1=<D1,D2,D5>, META2=<D1,D2,D4>, META3=<D1,D2,D4,D6>, META4=<D3,D5>, META5=<D3,D4>, META6=<D3,D4,D6>.

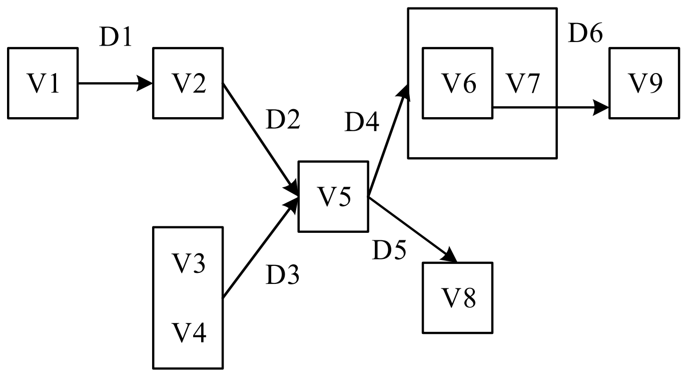


Figure 2: META diagram of supply chain

(2) Based on the constructed supply chain META diagram as well as the identification indicators, the comprehensive weights are determined in terms of both indicator weights and hierarchical weights.

By measuring both indicator weights and hierarchical weights, the final comprehensive weights can be determined as follows:

$$R_j^k = P_k \times Q_j (k = 1, \ldots, 5; j = 1, \ldots, 7) \tag{39}$$

(3) Selecting core enterprises based on the matrix calculation of supply chain META pathway

After standardizing the indicators in the core enterprise evaluation index system, combine the final weights to establish enterprise identification evaluation indexes. Set node $V_i (i = 1, ..., 9)$ in indicator $U_j (j = 1, ..., 7)$ under the indicator U_j^i. Assuming that each relative indicator in the initially selected core firm V5 is 1, the value of the node V_i under the indicator U_j can be obtained in the following equation:

$$U_j^i = \frac{U_j^i}{U_j^5} \quad (i = 1, ..., 9; j = 1, ..., 7) \tag{40}$$

From the relative indicators and comprehensive weights together to judge the level of different nodes under different indicators, to establish a comprehensive indicator system for identification and evaluation. Which calculates the benchmark value of each indicator in each layer, that is, to find out the optimal Z_j^i value of each layer of S2, S1, M, T1, T2. Since all the evaluation indicators are used in the positive evaluation indicators, the optimal value refers to the maximum value in $Z_j^i = U_j^i \times R_j^k$ for each layer.

For example, for pathway META1=<D1, D2, D5>, since it passes through nodes V1, V2, V5, and V8, the benchmark targeting gap of pathway META1 for metric U_i is shown in the following equation:

$$\begin{aligned} G_i^1 &= [Max(Z_i^1) - Z_i^1] \\ &+ [Max(Z_i^2, Z_i^3, Z_i^4) - Z_i^2] + (1-1) \\ &+ [Max(Z_i^6, Z_i^7, Z_i^8) - Z_i^8] \end{aligned} \tag{41}$$

Then for the whole pathway of META1 the benchmark targeting gap $G^1 = \sum_{i=1}^{7} G_i^1$. For all the META pathways, the pathway with the smallest benchmark targeting gap is calculated and selected as the optimal pathway. That is, $Z = Min(G^1, G^2, ..., G^6)$. Finally, the point with the best indicators among the nodes of the optimal pathway is the core firm node.

IV. Numerical analysis

In order to further explore the influence of different influencing factors on the stabilization strategy of the financing system of the supply chain financial platform, the interaction among core enterprises, banks and SMEs is studied. Specific parameters are set and Matlab is used to simulate the system evolutionary stabilization process for further verification and more intuitive analysis.

IV. A. Triangular Evolutionary Path of Financing

In order to further verify that the platform financing system can reach the evolutionary stability strategies represented by the three platform financing modes respectively when certain conditions are met, as well as to verify the validity of the evolutionary stability analysis. In this section, the initial parameter values are set to three groups, and the parameter values are substituted into the three-party evolutionary game model to simulate the evolutionary paths of the three platform financing modes using Matlab.

(1) Bank-enterprise cooperation supply chain finance platform

Let F_m (core enterprise platform construction cost) = 0.7, F_b (bank platform construction cost) = 0.7, F_a (banks' participation in the construction of the platform for banking-enterprise cooperation) = 0.4, λ (proportion of the proceeds received by the platform (the core enterprise) in the banking-enterprise cooperation mode) = 0.5, r_0 (interest rate of SME credit financing) = 0.05, r (interest rate of supply chain platform financing) = 0.3, α_m (rate of loss of opportunity cost of core enterprises) = 0.03, α_b (rate of loss of opportunity cost of banks) = 0.03, and d (demand for financing) = 6, I (SMEs' income) = 10, c (financing costs incurred by SMEs when they choose the "non-participation" strategy in favor of traditional credit financing) = 1, C (the cost of evaluating the supply chain-related information obtained by the banking platform) = 0.1. The stable solution of system evolution is (1,1,1), that is, the platform financing system eventually evolves to the core enterprises and banks to build a supply chain finance platform, SMEs participate in supply chain finance platform financing, stable point (1,1,1) of the system evolution path is shown in Figure 3.

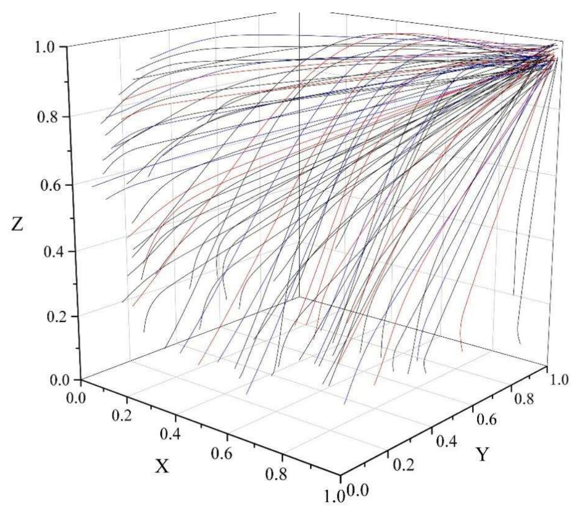


Figure 3: The System Evolution Path with ESS as (1,1,1)

(2) Core Enterprise Supply Chain Finance Platform

Let F_m =0.7, F_b =0.7, F_a =0.4, λ =0.3, r_0 =0.05, r =0.3, α_m =0.03, α_b =0.03, d =6, I =10, c =1, C =0.1. At this point, the system evolution stable solution is (1,0,1), that is, the platform financing system eventually evolves to the core enterprise builds supply chain finance platform, the bank does not build supply chain finance platform, and the small and medium-sized enterprise participates in the financing of supply chain finance platform. The system evolution path at the stable point (1,0,1) is shown in Figure 4.

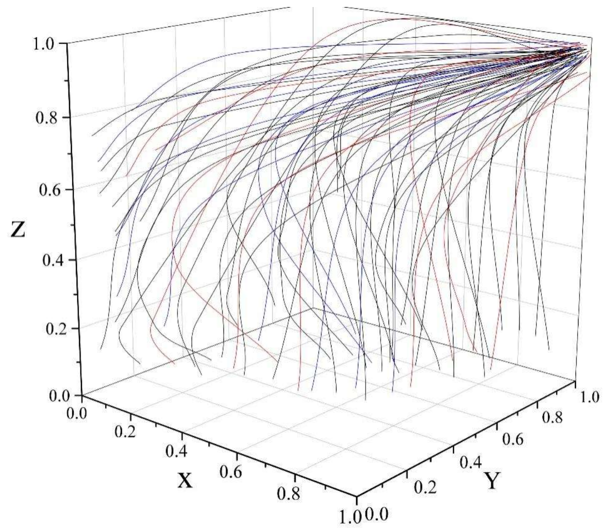


Figure 4: The System Evolution Path with ESS as (1,0,1)

(3) Bank Supply Chain Finance Platform

Let F_m =0.7, F_b =0.7, F_a =0.4, λ =0.5, r_0 =0.05, r =0.3, α_m =0.03, α_b =0.03, d =0.6, I =10, c =1, C =0.1. At this point, the system evolution stable solution is (0,1,1), that is, the platform financing system ultimately evolves to the core enterprise does not build supply chain finance platform, the bank builds a supply chain finance platform, and the small and medium-sized enterprise participates in the financing of supply chain finance platform. The system evolution path at the stable point (0,1,1) is shown in Figure 5.

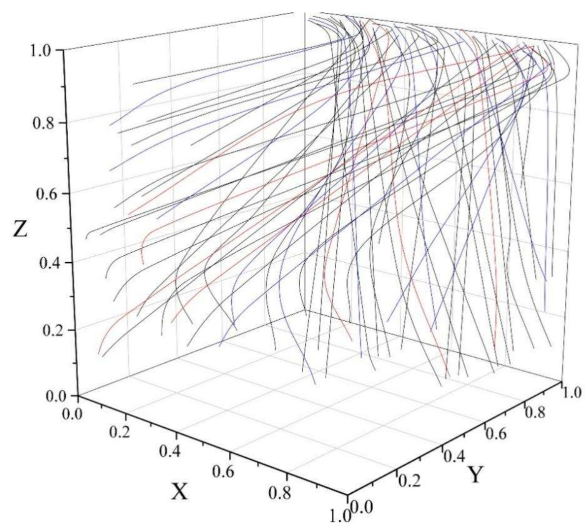


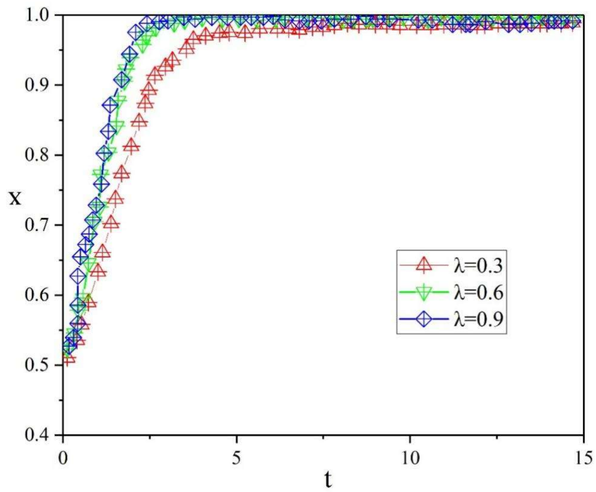
Figure 5: The System Evolution Path with ESS as (0,1,1)

IV. B. Impact of the share of proceeds on the evolution of the system

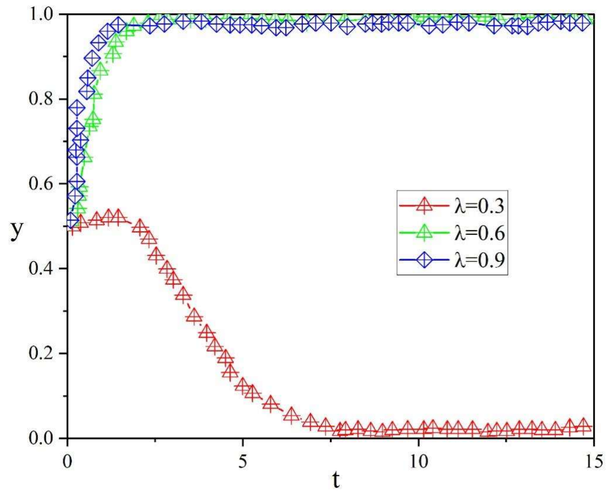
Analyze the effect of the distribution ratio of gains λ on the process and outcome of the evolutionary game, such that F_m = 0.7, F_b = 0.7, F_a = 0.4, λ = 0.5, r_0 = 0.05, r = 0.3, α_m =0.03, α_b =0.03, d =6, I =10, c =1, C =0.1. This is the initial scenario, and all other parameters are kept constant. The revenue allocation ratio λ is assigned to 0.3, 0.6, and 0.9, respectively, and the evolution of the system over time is shown in Figure 6.

Combined with the stability analysis, when the condition $d\left(r-r_0\right)-c<0$ is satisfied, the evolutionary stable strategy of SMEs is "participation", and the proportion of benefit distribution does not affect the strategy choice of SMEs.

Comparing Figures (a) and (b), when the revenue sharing ratio λ is small, the bank stabilizes to the "no-build" strategy, and the core enterprise stabilizes to the "build" strategy. The reason for this is that the smaller the revenue sharing ratio λ is, the smaller the revenue gained by the core enterprise and the larger the revenue gained by the bank under the bank-enterprise cooperation platform. The system eventually evolves into a model where the core enterprise builds the platform alone. This indicates that for the stronger core enterprises, when the cooperation to build a supply chain amount platform cannot get a higher allocation proportion for themselves, they will eventually choose to build a supply chain finance platform alone. In addition, observing Figures (a) and (b), it is found that when the cooperation can be reached, the higher the proportion of revenue distribution obtained by the core enterprises the faster the system evolves to the point (1,1,1).



(a) The evolutionary stability of the core enterprise



(b) The bank's evolutionary stability solution

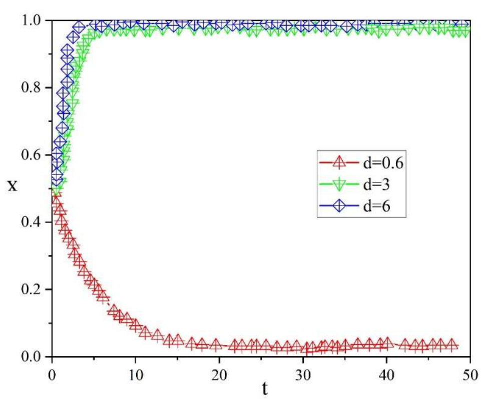
Figure 6: Influence of income distribution ratio on system evolution

IV. C. Impact of financing requirements on the evolution of the system

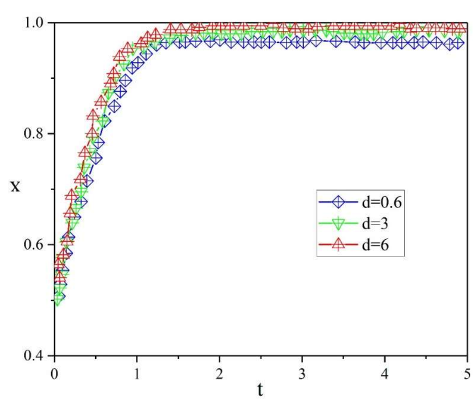
Analyze the effect of financing requirement d on the process and outcome of the evolutionary game such that F_m =0.7, F_b=0.7, F_a=0.4, λ=0.5, r_0=0.05, r=0.3, α_m=0.03, α_b=0.03, d=6, I=10, c=1, C=0.1. At this time, it is the initial scenario, and all other parameters are kept unchanged, and the financing demand d is assigned the values of 0.6, 3, and 6, respectively, and the results of the system evolution with time as shown in Figure 7.

Combined with the stability analysis above, when the condition $d(r-r_0)-c<0$ is satisfied, the evolutionary stable strategy of SMEs is “participation”, and the proportion of revenue distribution does not affect the strategy choice of SMEs.

Comparing Figures (a) and (b), when the amount of financing demand d is small, the core enterprise stabilizes to the “do not build” strategy, and the bank stabilizes to the “build” strategy. At this time, the bank assumes the responsibility of building a supply chain financial platform, in fact, the bank platform can also provide financing services for multiple, multi-industry supply chains. This shows that when the financing demand is small, the core enterprise is not suitable for building a supply chain finance platform and should not blindly develop online supply chain finance business.



(a) The evolutionary stability of the core enterprise



(b) The bank's evolutionary stability solution

Figure 7: The impact of financing demand on system evolution

IV. D. Impact of supplier fines on evolutionary outcomes

To carry out numerical simulation, take the parameter values C_2 (cost of acquiring information for the core business) = 1, C_3 (opportunity cost of using funds for the core business) = 2, A (principal amount of the loan) = 100, r_2 (interest rate of the loan for the core business) = 0.05, r' = 0.03, $R_2 = A(1+r_2)(1+r') = 110.36$. R_2, R_3 are the revenue gained from supplier's non-performance, and the penalties imposed by the core business on the supplier, respectively, and r' is the speculative interest rate. The initial values of p and q are set to discuss the impact of fines on suppliers on the evolutionary strategy. p represents the probability that the e-commerce firm chooses to take a loan, and q represents the probability that the supplier chooses to perform. R_3 represents the penalty that the supplier receives in case of default. The horizontal coordinate represents the probability that the e-commerce firm chooses to take a loan and the vertical coordinate represents the probability that the supplier chooses to perform.

The effect of R_3 on the evolution results is shown in Figure 8. When $R_3 > R_2 + Ar^2$, p, q both tend to 1. The larger the penalty, the faster it tends to 1. That is, both parties will choose the strategy of cooperation (loan, fulfillment). This is because in supply chain finance, the core enterprise effectively reduces the degree of information asymmetry due to the acquisition of transaction information and credit, while the supplier, due to the embedding of supply chain finance, the cost of default is much higher, and the performance is in line with the long term development, so the supplier credit in the e-commerce supply chain finance financing model will form a benign development.

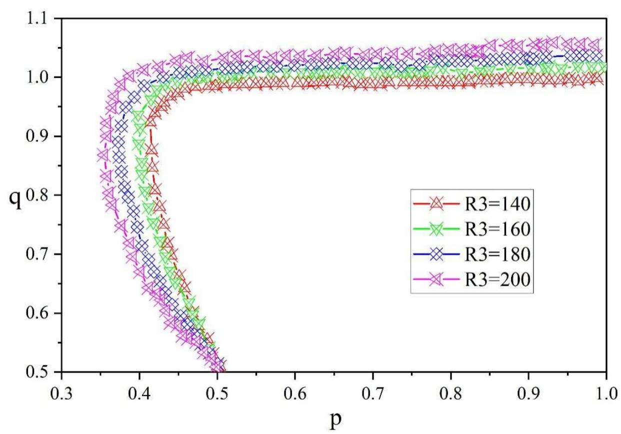


Figure 8: The effect of R3 on evolutionary results

The impact of R_3 on the evolutionary results is shown in Figure 9, when $R_3 < R_2 + Ar^2$, the whole system is in an extremely unstable state, no stable equilibrium strategy, this is because when the supplier's default cost is low, the supplier's gain from choosing to default is greater than the gain from performance after obtaining the credit, the Choose to default, at this time the core business will also find that the probability of supplier default becomes larger and change the probability of lending, that is, the core business of the loan is constantly changing with the supplier, which will lead to the risk borne by the core business becomes larger, and will lead to the vicious development of supplier credit.

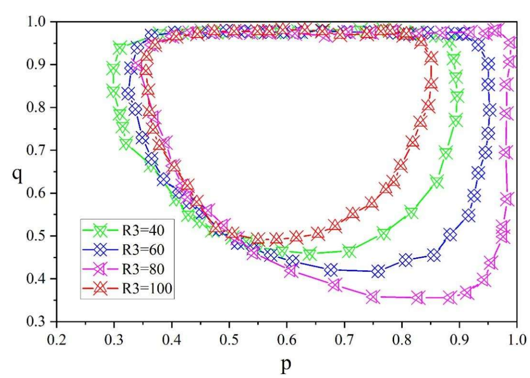


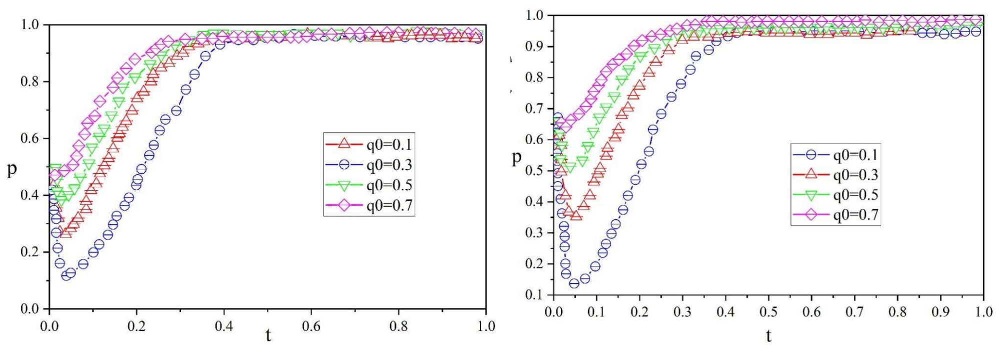
Figure 9: Evolutionary result

IV. E. Effect of initial probability on evolutionary outcomes

For numerical simulation, take the parameter values C_2 =1, C_3 =2, A =100, r_2 =0.05, $r^{'}$ =0.03, $R_2 = A(1+r_2)(1+r^{'}) = 115.74$. Change the initial value of q to discuss the impact of the initial probability of suppliers choosing “fulfillment” on the evolutionary strategy of core firms.

The effect of initial probability on the evolution outcome is shown in Figure 10. where q^0 represents the initial probability of the supplier choosing "performance", p represents the probability of the core enterprise choosing "loan", the abscissa represents the evolution time, and the vertical axis represents the probability of the core enterprise of the e-commerce platform choosing "loan".

As can be seen from the figure, the value of the initial probability that the supplier chooses “performance” affects the evolution of the core business loans and the speed of convergence of the curves. When the value of q^0 is small, that is, when the probability of the supplier choosing "fulfillment" is small, p will decrease in the direction of 0, because when the supplier defaults, the core enterprise will choose "no loan" in the subsequent strategy choice. p then converges to 1, because the penalty to the supplier is greater than the opportunity benefit brought by the default, the supplier will choose to perform to reduce the loss, at this time, the core enterprise will be willing to lend to the supplier, and finally converge to 1, and with the increase of q^0 the faster the curve converges to 1, that is, the greater the approximate choice of "performance" by the supplier, the faster the system can reach the stability point. When the initial value of p is changed to compare q^0 in taking the same value, the curve converges faster when the initial value of p is larger.



(a) p=0.5 (b) p=0.7

Figure 10: The effect of initial probability on evolutionary results

V. Conclusion

This paper analyzes the financing decisions of core firms in the order pledge model under supply chain finance conditions based on META graph theory. Consider the impact of default risk on the financing decisions of distributors, producers and banks.

(1) Divided into three financing modes: bank-enterprise cooperation supply chain finance, core enterprise supply chain finance, and bank supply chain finance, the system evolution stabilization solutions of the three financing modes are embodied as (1,1,1), (1,0,1), and (0,1,1), respectively.

When the revenue allocation ratio λ is smaller under the bank-enterprise cooperation platform, the smaller the revenue obtained by the core enterprise and the larger the revenue obtained by the bank. When the revenue sharing ratio λ is small, the core enterprise may favor the model of building the platform alone. However, when the financing demand d is small the core enterprise is not suitable for building a supply chain finance platform and should not blindly develop online supply chain finance business.

(2) In supply chain finance decision-making, all three parties play the game with the goal of maximizing their own profits. The supplier can choose randomly between the two strategies of “default” and “performance”. When $R_3 > R_2 + Ar^2$, p, q both converge to 1. The larger the penalty, the faster the convergence to 1. That is, both parties will choose a cooperative strategy (loan, performance). And $R_3 < R_2 + Ar^2$ when the whole system is in an extremely unstable state. When the supplier's default cost is low, it chooses to default, at which time the risk borne by the core enterprise becomes larger, which will result in the vicious development of the supplier's credit. Thus, the initial probability of suppliers choosing to “perform” will affect the evolution of the core enterprise loans and the speed of convergence of the curve.

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