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Exploring the Mathematical Properties of Eastern Pentatonic Scales and Western Pure Rhythmic Pitch Waveforms and Their Vocal Performance Differences Using Fourier Transforms

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Abstract In this study, the Fourier transform is used as the core method to systematically compare the mathematical properties of the Eastern pentatonic scale with the Western purely melodic pitch system and the differences in their vocal performances. The SWIPE algorithm is optimized by improving the Fourier transform to estimate the pitch of pentatonic scale. Based on the differentiated interval segmentation characteristics, the calculation method of the pitch score value of the pure law is deduced. Combining audio signal preprocessing and FFT algorithm, the harmonic structure and note recognition efficiency of the two types of meters are quantitatively analyzed. The study shows that the harmonic energy of the Eastern pentatonic scale is concentrated in the low-frequency band (50-200Hz) due to the mathematical characteristics of the five-degree phasing law, and the note intervals are concentrated in the interval of 0.6s~0.9s, and the note recognition accuracy reaches 99%. On the other hand, the western pure melody is based on the frequency proportionality of the natural overtone column, and the peak boundaries of the envelope are not obvious, which leads to significant high-frequency harmonic interference, and the note misdetection rate rises to 2.5%. This study reveals the deeper correlation between the mathematical properties of the metrical system and vocal performance through Fourier transforms, providing a quantitative analytical framework for cross-cultural music theory.

Index Terms Fourier transform, pitch estimation, tone difference, pure meter, pentatonic scale

I. Introduction

Traditional Chinese music theories often reveal the connotation of the modern concept of "scale" from different perspectives such as "tone", "rhythm", and "sound" [1], [2]. The pentatonic scale plays an important role in ancient Chinese music, as it is the foundation of ancient music, and is widely used in court music, folk music, and various musical performances [3], [4]. The pentatonic scale is characterized by the fact that it does not have chromatic (minor second) intervals, i.e., "pentatonic scale without chromatinics" or "diatonic pentatonic scale", often referred to as the "Chinese scale", and its pentatonic scale has special names in traditional Chinese culture, respectively called, gong, shang, jiao, zheng, and yu [5]-[8]. The rhythmic use of pentatonic scale is more flexible and diversified, and can be combined with different rhythmic patterns to create rich rhythms, which can be adapted to a variety of musical emotion expression needs [9], [10]. Pentatonic music often gives people a sense of simplicity and nature, close to people's lives, and easy to trigger emotional resonance [11]. Its music style has strong recognition, and is unique in the world of music [12], [13].

Pure law is a system of intonation and tuning that generates all other intervals in natural fifths and thirds, and it originated in Europe [14], [15]. Pure law is a metrical system generated on the basis of overtone columns [16]. It is based on natural harmonics and determines pitch based on the frequency ratio of natural overtones produced by string vibrations [17]. Early in the development of music, through the observation and practice of stringed instruments, people found that the vibration of the strings would produce a series of harmonic overtones, and based on the frequency relationship of these overtones gradually formed the pure meter [18]-[20]. In ancient Western music, pure meter was the only accepted system of music theory, which constructed different intervals and modes based on integer ratios [21], [22]. Compared with the pentatonic scale, the vocal performance of the two differed in terms of melodic lines, rhythmic characteristics, and harmonic use [23], [24].

In this paper, we first sort out the common preprocessing methods for audio signal analysis and propose to use FFT to mine the pitch waveform of audio signals. The source streams of pentatonic scale and pure meter are elucidated, and the improved SWIPE algorithm is adopted to estimate the pitch of the oriental pentatonic scale.

Dynamically adjusting the window function with linear combination of pitch intensity curves by improved Fourier transform. Based on the frequency proportionality of natural overtone columns, the interval segmentation characteristics of Western pure melodies are quantified and the calculation method of tone scores is deduced. Relying on FFT, quantitative comparison of note recognition features of two types of scales is conducted to explore the differences in their vocal performance.

II. Overview of audio signal analysis techniques

II. A.Audio signal pre-processing

Audio signal preprocessing usually includes operations such as slicing, pre-emphasis, framing, and windowing. The commonly used audio signal preprocessing operations are introduced below.

(1) Slicing

Slicing refers to the operation of dividing an audio file into multiple segments by an equal length of time. On the one hand, slicing the audio file is to keep the time length of the analyzed audio signals consistent, which is convenient for unified processing; on the other hand, it can increase the number of input samples, which is also a kind of data enhancement operation from a certain point of view, which is conducive to the training of a better classification model and improves the model's generalization ability. Studies have shown that segment-based feature extraction is more effective than whole song-based feature extraction in music classification tasks.

(2) Pre-emphasis

By pre-emphasizing the audio signal, the high-frequency component of the audio signal can be boosted to reduce the interference caused by the vocal cords and lips during speech vocalization. Pre-emphasis is generally realized by a first-order high-pass filter, the transfer function of which is shown in equation (1).

$$H(z) = 1 - \mu z^{-1} \tag{1}$$

where μ denotes the pre-emphasis coefficient, which generally takes a value between 0.9 and 1.0, and is usually taken as $\mu = 0.97$.

(3) Framing

The audio signal is generated by the regular vibration of the sound emitting body, which is easily affected by the environment and is a kind of non-smooth signal. However, in a shorter time range, the audio signal can be considered as a smooth signal. For example, the spectral characteristics of an audio signal remain essentially unchanged over 10-50 milliseconds. Considering the continuity of the signals of two adjacent frames, in order to avoid jumps and discontinuities in the signals between frames, overlapping frames are generally used, i.e., there is a certain overlap between two adjacent frames, which is called frame shift. The frame shift is generally 1/3 or 1/2 of the frame length.

(4) Add window

Adding a window is essentially a small segment of the frame is applied to a certain operation, the signal is smoothed, usually immediately after the frame operation. Commonly used window functions include rectangular window and Hamming window.

The expression for rectangular window is as follows:

$$\omega(n) = \begin{cases} 1 & 0 \le n \le L-1 \\ 0 & \text{Other} \end{cases} \tag{2}$$

The expression for the Hamming window is as follows:

$$\omega(n) = \begin{cases} 0.54 - 0.46\cos\left(\dfrac{2\pi n}{L-1}\right) & 0 \le n \le L-1 \\ 0 & \text{Other} \end{cases} \tag{3}$$

Different window functions have different effects on the signal spectrum. For example, the rectangular window is abruptly truncated at both ends of the signal, resulting in high-frequency interference and spectral leakage brought about by the windowing process. The Hamming window, on the other hand, can effectively suppress the spectral leakage while maintaining the high-frequency characteristics of the signal.

II. B.Fast Fourier Transform

The Fourier transform can convert any time series signal into an infinite accumulation of sine wave signals of different frequencies. The intensity of the sinusoidal signals at different frequencies represents the intensity of the signal at that frequency, thus extracting the frequency domain information from the time domain signal to achieve the purpose of analyzing the signal from a new perspective and further mining information. Discrete Fourier Transform (DFT) is a Fourier transform that discretizes the frequency domain, allowing computers to process information in the frequency domain.

For a time-domain discrete signal $x[n]$ of length N, the formula for its discrete Fourier transform is as follows:

$$X(e^{j\omega}) = X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}, \left(k = 0, \cdots, N-1\right) \tag{4}$$

When dealing with digital audio, we are oriented to the data object is a digital signal, that is, a series of discrete finite-length data points, if you want to time-domain signals into the frequency domain representation, from the point of view of the Fourier transform we need to use the discrete Fourier transform. If it is calculated directly, its calculation is relatively large.

Fast Fourier Transform (FFT) has a significant position in the field of digital signal processing. Fast Fourier Transform is a fast and efficient algorithm to compute the discrete Fourier Transform.The computational complexity of DFT is $O(N^2)$, while FFT utilizes the symmetric nature and periodicity of DFT to reduce the computational complexity to $O(Nlog_2 N)$. It divides a sequence of length N into two subsequences of length $\frac{N}{2}$, corresponding to even and odd sequences, and then recursively performs the same operation on the subsequences until the length is 1. This reduces the amount of computation and increases efficiency.

Cooley-Tukey, a common algorithm for implementing the Fast Fourier Transform, requires that the number of input signal points must be a power of two. If this requirement is not met, zeros can be added after the signal. From the point of view of the discrete Fourier transform, the added zeros will not affect the frequency domain of the signal, but will only improve the resolution of the FFT and increase the density of the spectral data.

Taking the signal $x[n]$ of length N as an example, the discrete Fourier transform $X\left(e^{j\omega}\right) = \sum_{n=0}^{N-1} x[n] e^{-j\omega n}$ is shown in Eqn. (4), with $\omega = 0, \frac{2\pi}{N}, \frac{4\pi}{N}, \ldots, 2\pi$, and if the complementary zeros are complementary to a length of N_l, one can think of the complementary zeros as a superposition of $x[n]$ and the all-zero signals $z[m]$ of a length of N_l, where $m = \left(0, 1\, ldots N_l - 1\right)$, so the signal with complementary zeros is written as $y[n] = \begin{cases} x[n], & 0 \le n < N \\ z[n], & N \le n < N_1 \end{cases}$, whose discrete Fourier transform is $Y(e^{j\omega}) = \sum_{n=0}^{N_1-1} y[n] e^{-j\omega n} = \sum_{n=0}^{N-1} x[n] e^{-j\omega n} + \sum_{N}^{N_1-1} z[n] e^{-j\omega n} = \sum_{n=0}^{N-1} x[n] e^{-j\omega n}$ $\omega = 0, \frac{2\pi}{N_1}, \frac{4\pi}{N_1}, \cdots, 2\pi$. As shown in Figure 1, we can see that the discrete Fourier transform of $y[n]$ is only higher in resolution and denser in data points than the discrete Fourier transform of $x\left[n\right]$ before zeroing.

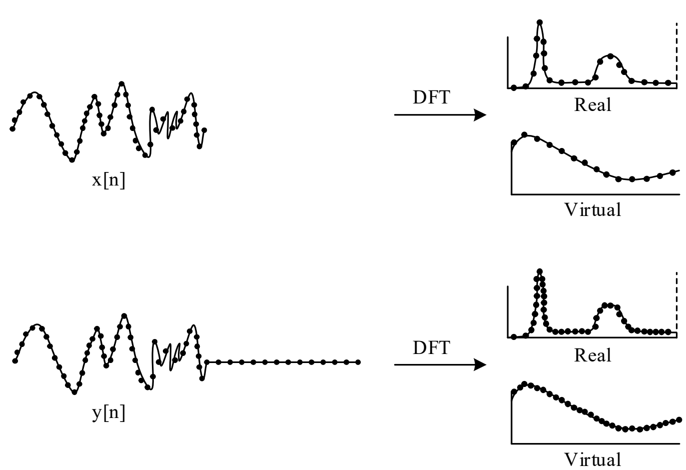

Figure 1: Comparison of DFT before and after zero filling

The key role of the Cooley-Tukey algorithm is to reduce the computational complexity of polynomials, and is a typical example of a partitioning algorithm. Take the polynomial $y\left(x\right)$ as an example:

$$y(x)=\sum_{i=1}^{n}a_i x^i=a_0x^0+a_1x^1+a_2x^2+\cdots+a_{n-1}x^{n-1}+a_nx^n \tag{5}$$

First the polynomial is expanded to 2^k terms and the total number of terms must end up being greater than n. Then by parity count we can convert $y(x)$ into the sum of two polynomials with the same number of terms:

$$y(x)=y_0(x)+y_1(x) \tag{6}$$

Among them:

$$y_0(x)=\sum_{i=0}^{\frac{n}{2}-1}a_{2i}x^{2i}=a_0x^0+a_2x^2+\cdots+a_nx^n \tag{7}$$

$$y_1(x)=\sum_{i=0}^{\frac{n}{2}-1}a_{2i+1}x^{2i+1}=x\sum_{i=0}^{\frac{n}{2}-1}a_{2i+1}x^{2i}=a_1x^1+a_3x^3+\cdots+a_{n-1}x^{n-1} \tag{8}$$

Substituting the n th unit root ω_n^k into $y_0(x)$ and $y_1(x)$, i.e., letting $x=\omega_n^k$, and $k=0,1,2,\cdots,n-1$, yields:

$$y_0(\omega_n^k)=\sum_{i=0}^{\frac{n}{2}-1}a_{2i}\omega_n^{2ki}=\sum_{i=0}^{\frac{n}{2}-1}a_{2i}\omega_{\frac{n}{2}}^{ki} \tag{9}$$

$$y_1(\omega_n^k)=\sum_{i=0}^{\frac{n}{2}-1}a_{2i+1}\omega_n^{2ki+1}=\omega_n^k\sum_{i=0}^{\frac{n}{2}-1}a_{2i+1}\omega_n^{2ki}=\omega_n^k\sum_{i=0}^{\frac{n}{2}-1}a_{2i+1}\omega_{\frac{n}{2}}^{ki} \tag{10}$$

So:

$$y(\omega_n^k)=\sum_{i=0}^{\frac{n}{2}-1}a_{2i}\omega_{\frac{n}{2}}^{ki}+\omega_n^k\sum_{i=0}^{\frac{n}{2}-1}a_{2i+1}\omega_{\frac{n}{2}}^{ki} \tag{11}$$

Up to this point, it can be seen that the equation that would have required n substitutions now requires only $\frac{n}{2}$ times. Using the recursive algorithm, the corresponding coefficients can finally be found in $O(1)$ complexity after $log_2 n$ times of recursion. In other words, this paper successfully reduces the computational complexity from $O(n^2)$ to $O(nlog_2n)$.

III. Application of Fourier Transforms in the Analysis of Eastern Pentatonic Scale and Western Pure Rhythmic Vocal Music

The Eastern and Western music systems form unique aesthetic expressions due to the differences in rhythmic systems: the Eastern pentatonic scale is based on the law of fifths, generating scales through the superposition of pure fifth intervals, and its harmonic structure presents simplicity in traditional East Asian instruments (e.g., the guqin and guzheng). Western pure law, on the other hand, is based on the frequency proportionality of natural overtone columns, emphasizing the concordance of third and sixth intervals, and forming a polyphonic system with a differentiated division between whole tones and semitones. However, most of the existing studies focus on qualitative descriptions of musicology and history, and lack quantitative comparisons of the mathematical and acoustic characteristics of the two types of metrical systems. Based on the Fourier transform theory, this study systematically explores the mathematical properties and acoustic performance differences between the Eastern pentatonic scale and the Western pure metric pitch system.

III. A. Eastern Pentatonic Scale and Western Pure Law

III. A. 1) Pentatonic scale

A pentatonic scale is a scale with five steps within an octave. It is found in almost all peoples and regions of the world, and has always played an important role in the thousands of years of human musical development. In the history of Chinese music, the pentatonic scale is definitely the main core of existence, and at the same time, the pentatonic scale has a strong influence in many regions of the world. The pentatonic scale is not only found in China, but also in almost the whole of Southeast Asia, Northeast Asia, South Asian subcontinent, Oceania, and Turkey, and even the whole of the Arabian countries, and it is commonly found in the Eurasian nomadic peoples, African blacks and Berbers, American Indians, European Scots, and English Celts. At the same time, the pentatonic scale of all these regions also exists in greater or lesser differences, because the pentatonic scale is not a kind of scale, all the scales composed of five different pitch levels are called pentatonic scale, these pentatonic scales have different structures, showing different musical philosophies.

III. A. 2) Pure law

Pure rhythm first originated in Europe. In Europe, before the 9th century, it was monophonic music, and choral weaving was basically non-existent. After the 9th century, polyphonic music began to sprout, and in the early days polyphonic music only used a combination of octaves, pure fifths and pure fourths, and simple choral weaves appeared.

At the beginning of the 14th century, with the development of polyphonic music, the simultaneous combination of third and sixth intervals was widely used, and the fifths could not satisfy the needs of polyphonic music, so the chorus formally began to come into its own in terms of consonance, thus promoting the application of pure meter. During this period, many music theorists noticed the intervals of pure meter, and combined them with polyphonic music to obtain harmonious sound effects. Based on the theory of pure meter advocated by the Greek Harmonists, the triadic interval of pure meter was formally proposed between 1275 and 1300, and the intervals of the triad and the hexadecimal interval were theoretically combined to form harmonic intervals. The intervals of pure melody are the major and minor thirds and major and minor sixths of pure melody, and the proposal of pure melodic intervals made people further realize the law of the interval system, and laid a theoretical foundation for the wide application of pure melody in the Renaissance period.

In Europe, from the 15th to the 17th centuries, tonal music was produced and gradually matured, and pure melody was widely used in choral music, and an a cappella theme became more and more popular, which is also called the “pure melody period”. This period is also known as the “Pure Melodic Period”. The introduction of pure melodic major scales and chord principles laid the theoretical foundation for the development of harmonic theory.

III. B. SWIPE-based Pentatonic Scale Pitch Estimation

The Sawtooth Waveform Excitation Pitch Estimation Algorithm (SWIPE) was chosen to estimate the pitch of the Eastern pentatonic scale. Most of the errors caused by the pitch estimation algorithm are not random: they consist of octave or suboctave estimations of the pitch as pitch. The most common fundamental pitch estimation errors are shown in Figure 2.

In Figure 2, B represents the fraction that uses both the positive and negative cosine parts, and the effect on the 200 Hz peak is clear: it has disappeared. a obtains the same effect for higher-order multiples of 100 Hz. c represents the fraction for which only the positive cosine part is used as a kernel, for each candidate pitch. d and e represent the positive cosine half-periods (consecutive curves) identifying the pitches of 50 and 200 Hz, and the negative cosine half-periods in between them, respectively. The positive cosine curves of the 50Hz/200Hz harmonics make a positive contribution to the 50Hz/200Hz candidate pitch scores, but this is offset by the negative cosine curves at the odd octave of 25Hz/100Hz.

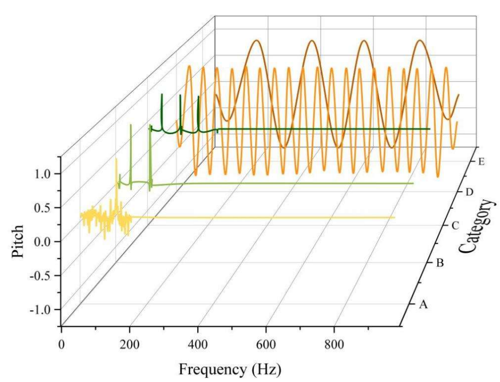

Figure 2: Most common pitch estimation errors

The effect of using negative weights for pitch overharmonics is shown in Figure 3. It represents the fundamental frequency in the 100Hz signal spectrum, as well as all harmonics at the same amplitude (vertical line). (Only harmonics up to 1kHz are shown, but the signal contains harmonics up to 5kHz) To improve visualization, the elements are shown as lines, but usually they are wider and the width depends on the window size.

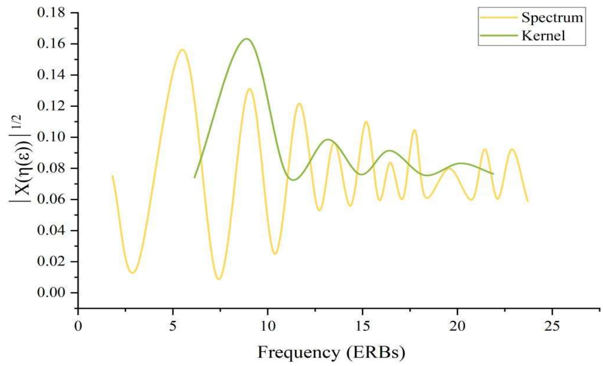

Figure 3: SWIPE' kernel

The computation of Fourier transform is the most computationally expensive operation for SWIPE and SWIPE', so it is important to reduce the number of Fourier transforms in order to minimize the computational cost.

In this paper, the 2nd dense (P2) WS is used to replace the O-WS, and the pitch intensity curve is shown in Fig. 4. Unfortunately, this method produces discontinuities in the pitch strength (PS) curve. To emphasize the effect, the signal pitch (220 Hz) was chosen as the point at which the WS changed. Because a large window in the region adjacent to the pitch produces a larger PS value than that produced by a small window, the pitch may be biased toward the lower value.

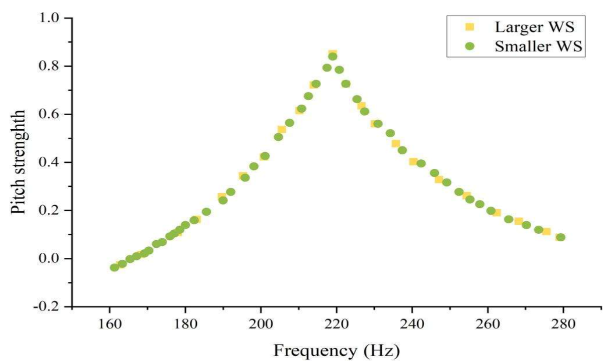

Figure 4: Individual pitch strenghth curves

But in this paper, a better solution is found: the PS is computed as a linear combination of the PS values generated by the two nearest PS-WSs, with a combination coefficient proportional to the logarithmic distance between the P2-WSs and the O-WS.

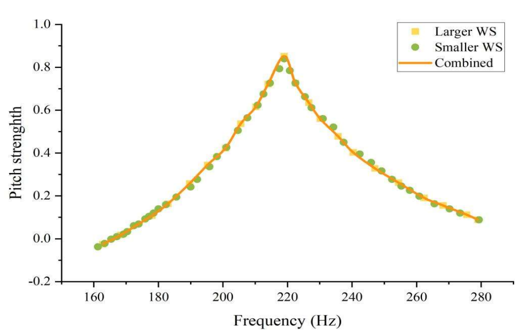

Figure 5: Combined pitch strength curves

Specifically, to determine the PS value used to compute a candidate with a frequency of f Hz, P2-WSs, O-WS is written as a dense number of times 2, $N^* = 2^{L+\lambda}$, where L is an integer and $0 \le \lambda \le 1$. Then, PS values $S_0(f)$ and $S_1(f)$ are computed using windows of size 2^L and 2^{L+1}, respectively. Finally, these are combined by the PS values to produce the final PS.

$$S(f)=(1-\lambda)S_0(f)+\lambda S_1(f) \tag{12}$$

The combination of PS curves is utilized to smooth the intermittent points appearing in Fig. 4, and the results are shown in Fig. 5, thus realizing the estimation of the Oriental pentatonic scale.

III. C. Purely metrical tone difference analysis based on data derivation

The method of pure law is based on the natural overtone system, and the difference between two neighboring tones can be calculated by knowing the value of each tone in the pure law major scale, e.g., the value of the tone of c1 is 0, the value of the tone of d1 is 204, and the value of the tone of c1 is subtracted from the value of the tone of d1 to arrive at the transverse difference of the two tones, i.e., the difference between two adjacent tones can be calculated as the transverse difference between two tones, i.e., the difference between two neighboring tones can be calculated as the difference between two neighboring tones, and the difference between two neighboring tones can be calculated as the transverse difference between two neighboring tones. The difference between the two tones is the transverse difference between the two tones, from which the difference between the remaining transverse neighboring tones can be calculated.

The results of the calculation of the transverse tone difference of the pure major scale are shown in Table 1. The difference between the two tones can be summarized as two kinds of whole tones in the pure law: one is the big whole tone, with a score of 204, and the other is the small whole tone, with a score of 182, and the semitone, with a score of 112, is called the "big semitone". The intervals between two neighboring tones in a purely metrical scale can be determined by calculating the difference between the two tones in the following order: major whole tone, minor whole tone, major semitone, major whole tone, minor whole tone, major whole tone, major semitone, major semitone.

Table 1: Results of transverse tone difference calculation

Sound level	1	2	3	4	5	6	7	8
Phonetic name	c1	d1	e1	f1	g1	a1	b1	c2
Sound score	0	204	377	482	698	872	1076	1185
Lateral pitch difference	204	173	105	216	174	204	109	

Purely metrical major scales are routinely limited to the key of c, and no specific data can be provided for reference when the work deals with other tonalities. The intervals between neighboring tones in the c scale can be used to derive tone scores for tones in other keys. For example, if an a cappella work is in the key of bA major, the intervals between neighboring tones in the pure melodic major scale can be used to derive its pure melodic scale, and the pure melodic scale of bA major is shown in Table 2. The purely metrical scale has an octave difference of 1185, and when the melody is more than one octave up, 1185 cents can be added to the cents of the tone in the octave to obtain the corresponding cents.

Table 2: Pure temperament scales in bA major

Sound level	1	2	3	4	5	6	7	8
Phonetic name	ba1	bb1	c2	bd2	be2	f2	g2	ba2
Sound score	0	204	377	482	698	872	1076	1185
Relation of intervals	Major tone	Minor tone	Major semit	Major tone	Minor tone	Major tone	Major semit	

The calculated score values for each tone are shown in Table 3. Pure law major scale is the central C dominant upward constitutes the upper octave of the scale, for more than one octave down the tone score value, in accordance with the pure law major scale in the order of the relationship between the two neighboring tones, reverse projection, if the tone score value of c1 as 0, according to the interval relationship between the two neighboring tones in the upper scale downward projection, downward, will produce a negative value of the representation.

Table 3: Calculation of the score value of a tone more than one octave down

Sound level	1	2	3	4	5	6	7	8
Phonetic name	c1	B	A	G	F	E	D	C
Sound score	0	-109	-313	-487	-703	-808	-981	-1185
Relation of intervals	Major semit	Major tone	Minor tone	Major tone	Major semit	Minor tone	Major tone	

III. D. Analysis of the differences between Eastern pentatonic scales and Western purely rhythmic vocal performance

Two guqin pieces, three guzheng pieces, two operas, and three masses were selected for analysis, numbered A~J, and divided into two music files based on the categories, to explore the differences in vocal performance between the Eastern pentatonic scale and the Western pure meter.

Firstly, the music was subjected to feature extraction. It is found that the note information of music is mainly contained in the low-frequency components from 50Hz to 200Hz, so the extraction of notes and note features can be carried out by analyzing the low-frequency components in music. Using Fourier transform to amplify the high frequency part of the audio signal, take each of its local maximum points as the peak point, then these points are likely to be the location of the note. After getting the positions of all peak points by peak detection, the note points can be obtained by using statistical methods according to the consistency of the note time interval. First assume that all the peak points are the starting note points, traverse all the starting note points, calculate the time interval of each two notes, make statistics, and the statistical results are shown in Figure 6. As can be seen from Fig. 6, the note interval of the Eastern pentatonic scale file is mainly concentrated in the interval of 0.6s~0.9s, and the note interval of the Western pure meter file is mainly concentrated in the interval of 0.4s~0.7s. Iterate through the peak point, if the point is the peak point, and the time interval with the previous note is on the centralized interval, then the point is the starting point, otherwise it is the error point. After the starting point of the note is extracted, then the basic characteristics of the note such as note length, pitch, and intensity can be extracted on this basis.

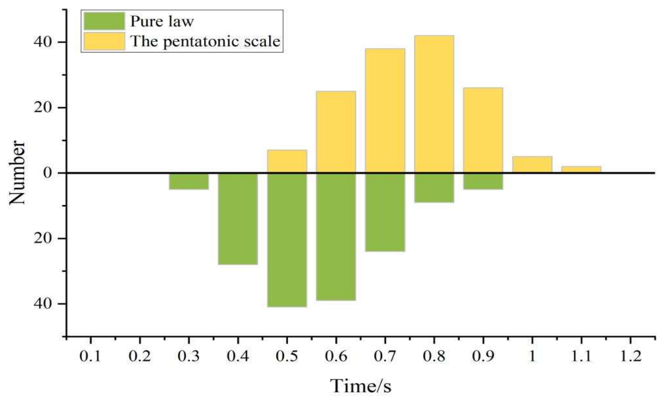

Figure 6: Note interval statistics

The note and bar line positions of each piece of music are manually labeled through the music's score. The results of the Fourier transform-based note extraction results and the hand-labeled note position information are compared, and the results are shown in Table 4. For the Eastern pentatonic scale, the demarcation line between the notes of the low-frequency signal is clearer, and the peaks of the envelopes are easier to extract, so the recognition rate of the notes is higher, and the recognition accuracy reaches 99%. As for the western pure meter, the peak boundaries of the envelope are not obvious, so the recognition rate of the notes is lower, with a recognition accuracy of 97.5%, and the number of false detections and omissions is higher. However, it still has a high recognition rate overall.

Table 4: Results of different types of musical notes extraction

Music	Category	Musical length	Actual note count	Detected note count	The number of false notes	Missing note count
A	The Oriental pentatonic scale	79s	86	86	0	0
B	The Oriental pentatonic scale	62s	71	71	1	0
C	The Oriental pentatonic scale	78s	83	83	1	1
D	The Oriental pentatonic scale	54s	62	62	0	0
E	The Oriental pentatonic scale	75s	85	84	0	1
F	Western pure law	66s	74	72	0	2
G	Western pure law	57s	68	68	3	3
H	Western pure law	61s	72	71	2	3
I	Western pure law	48s	55	55	0	0
J	Western pure law	72s	80	79	0	1

IV. Conclusion

This paper first examines the mathematical properties of the Eastern pentatonic scale and the Western pure melodic pitch, and then analyzes their note characteristics based on the Fourier Transform to explore the deeper connection between the mathematical properties and the differences in vocal performance.

(1) Analysis of mathematical properties of pitch

Oriental Pentatonic Scale: The mathematical characteristics of the pentatonic scale result in a high concentration of harmonic energy in the low frequency band (50-200Hz). There is no semitone division within the scale, the frequency ratio between neighboring levels is fixed, and the spectral envelope is smooth, providing clear features for peak detection by Fourier transform.

Western pure law: the frequency ratio of natural overtone columns forms a differentiated interval division, and such division leads to the dispersion of harmonic energy, increasing the complexity of pitch estimation.

(2) Comparison of vocal performance results

Eastern pentatonic scale: note intervals are mainly concentrated in the interval of 0.6s~0.9s, the demarcation line between the notes of low-frequency signals is clearer, the peak value of the envelope is easier to be extracted, and the note recognition accuracy reaches 99%.

Western pure meter: the note interval is mainly concentrated in the interval of 0.4s~0.7s, the peak boundary of the envelope is not obvious, and the note misidentification rate increases to 2.5%.

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