

Using Data Envelopment Analysis to Explore Differences and Convergences in Regional Real Estate Markets

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Abstract In this paper, regional real estate evaluation indexes, real estate input and output efficiencies are sorted out. Under the condition of constant returns to scale, a three-stage DEA method is used to equalize the relative efficiencies of decision-making units in multiple-input and multiple-output problems, and the three-stage DEA method is combined with the Malmquist index method to form a Malmquist-DEA dynamic evaluation model, which in turn reflects the changes in total factor productivity over successive periods. The spatial autocorrelation method and Markov chain are used to explore the patterns and trends in the distribution of geospatial data variables. The results show that the overall total factor productivity index and technical efficiency increased by 2.77% and 3.05% on average, while the technical progress index decreased by 0.27%. The increase in the technical efficiency index is the main reason for the increase in total factor productivity in China's real estate industry. The total factor productivity index as a whole shows that eastern > central > western regions, similar to the economic development situation. The total factor productivity of different provinces shows strong regional differences, and real estate development is affected by neighboring cities in the short term, making it difficult to achieve transfer across stage levels, and the degree of risk of transferring to a low level state is low.

Index Terms Three-stage DEA method, Malmquist index, Spatial autocorrelation method, Markov chain, Real estate

I. Introduction

Real estate industry is an important basic industry for the development of the national economy, with high industrial correlation, which is the leading force to promote the development of the whole national economy and accelerate the realization of industrialization and urbanization [1], [2]. At the same time, real estate, as one of the clothing, food, housing and transportation, is an important part of the asset portfolio of residents' families, and has an important impact on the standard of living of residents' families [3], [4]. The healthy and orderly development of real estate industry has an important impact on the economic operation of the whole country. On the one hand, the development of real estate quickly drives the development of other industries with its multiplier effect to bring economic prosperity, on the other hand, once the real estate crisis occurs, its multiplier effect will in turn bring severe damage to the economy [5]-[8].

As real estate is naturally a regional market, the promotion of real estate market development can not take a one-size-fits-all form, but should analyze the characteristics of different regional real estate market, to take differentiated development strategies [9]. In terms of reality, whether it is a different province or province of the real estate market in terms of investment, sales, prices, employment and other aspects of the real estate market in different regional markets there are large differences in the real estate market in the main problems are also different [10]-[12]. The rapid development of real estate market in the first and second tier cities, the price of housing is daunting, housing difficulties are becoming increasingly serious, the third and fourth tier cities, the degree of development of real estate is low, the effective demand is insufficient, the vacancy rate is abnormally high and even the phenomenon of "ghost towns" [13]-[15]. The difference between the real estate in different regions is very obvious, and the impact of different factors on the development of real estate in different regions will also be different [16]. Therefore, pay close attention to the differences and convergent changes in the real estate market has become the core of the whole real estate industry research.

Some scholars have carried out research on regional differences and convergence of real estate economic development. Literature [17] carried out the analysis of geographical differences in regional real estate market investment in the context of population aging, and the empirical investigation found that the strength of urban real estate investment has a close connection with the level of regional economic development, and showed large

regional differences in the results. Literature [18] established a geostatistical model with the number of normalized transactions (NNT) registered in the real estate market as the data, and used the method of spatio-temporal analysis to describe the evolution of the regional real estate market with the changes in the economic cycle, and at the same time revealed the differences in the spatial distribution of real estate market. Literature [19] investigated the spatial differences in real estate prices and asset liquidity, and effectively proved the location effect of the real estate market by evaluating the differences in real estate sales prices and asking prices under different locations, as well as the relationship between market liquidity and geographic location. Literature [20] proposed to use clustering algorithms to cluster regions based on price and liquidity in real estate markets, and found that users belonging to the same cluster always have the same demographic characteristics, as well as showing similar knowledge and methods in real estate investment. Literature [21] used principal component analysis to segment the direct real estate market in the United Kingdom and pointed out that there is a significant convergence of average monthly returns and yields in the regional real estate market, which effectively assesses the return on investment in the regional real estate market. Literature [22] used the data envelopment method to empirically analyze the supply-side efficiency of the real estate market, indicating that cities in different geographic locations and economic development status have different performance in the supply-side efficiency of the real estate market, and the relevant results can help to improve the development efficiency of the real estate market, and at the same time, promote sustainable urbanization.

In this paper, in order to study the differences and convergence of regional real estate markets, we propose to measure their input-output efficiency and total factor production efficiency based on the relaxation data envelopment analysis (DEA) method. In view of the fact that it is difficult to effectively solve the practical and complex problem of multiple inputs and outputs in the regional real estate industry by using a single model, this paper uses the decision unit (DUM) to combine the Malmquist index with the data envelopment analysis (DEA) method to obtain a dynamic evaluation model based on Malmquist-DEA, and changes the proportion of output factors to make its output level consistent with the decision-making unit at the forefront of production. The global Moran index and the local Moran index are then used to measure the spatial autocorrelation within the entire spatial extent and between each spatial unit and its neighboring spatial unit variables. Finally, the differences and convergence of regional real estate markets are analyzed.

II. Malmquist-DEA-based methodology for regional real estate market analysis

II. A. Evaluation of input-output efficiency and total factor productivity in the real estate industry

II. A. 1) Principles for the selection of indicators

In the use of relaxation-based data envelopment analysis (DEA) method [23], [24] for efficiency measurement, the indicators can not be arbitrarily selected and used, the selection of input-output indicators should follow the importance of the time, diversity, comparability and feasibility of the four principles, the specific principles are as follows:

(1) Importance principle

The purpose of indicator selection is to truly reflect the essence of the problem, and must be representative.

(2) Diversity principle

Selection of indicators in accordance with the principle of importance at the same time do not ignore the diversity of indicators, neither can there be no center, nor too one-sided, select multiple input indicators and output indicators to measure efficiency.

(3) Principle of Comparability

In order to help the evaluation and comparison of data, and also to facilitate data calculation and validation, it should be committed to ensuring that each input indicator and output indicator is consistent in terms of the content, unit, etc. of the calculation.

(4) Principle of feasibility

The selected indicators must ensure that the corresponding data information can be obtained and that the reliability and simplicity of data collection is ensured.

II. A. 2) Selection of indicators

Because of the uniqueness of China's real estate, this paper selects the following indicators to establish the real estate industry efficiency evaluation system. The input and output indicators for real estate industry efficiency evaluation in this paper are shown in Table 1.

Table 1: This article measures the efficiency of the real estate industry

Categories	Indicator	Index name
Input index	Labor investment	The number of people in the real estate industry
	Capital investment	Annual investment
Output indicator	Scale output	Commercial housing
	Economic output	Commercial housing sales
		Rental income
		External income

II. A. 3) Three-stage DEA approach

(1) Parametric frontier analysis

The parametric frontier analysis method requires the construction of a definite production function to express the relationship between inputs and outputs to each other, and the use of regression to specify the parameters in this expression based on one of the sets of input and output data under the constraints.

(2) Non-parametric frontier analysis

DEA is the most commonly used non-parametric frontier analysis.

(3) Combination of parametric and non-parametric analysis: three-stage DEA approach

This paper combines parametric and non-parametric analysis methods to evaluate the input-output efficiency of real estate industry in each province. The three-stage DEA methods are:

a) First-stage DEA: calculating initial efficiency and slack variables of input variables

(a) DEA-CCR model

The DEA-CCR model can evaluate the relative efficiency of decision-making units in the problem of “multiple inputs and multiple outputs” under the condition of constant returns to scale.

Suppose there are n decision-making units, denoted as $DMU_j (j=1,2,\dots,n)$, each of which has m types of inputs and s types of outputs, where the input vectors are set as $x_j(x_{1j},x_{2j},\dots,x_{mj})^T > 0, j=1,2,\dots,n$ and the output vector is set to $y_j(y_{1j},y_{2j},\dots,y_{sj})^T > 0, j=1,2,\dots,n$. Denote by x_{ij} the amount of output of the j th decision unit for the i th type ($i=1,2,\dots,m$), and by y_{rj} the amount of output of the j th decision unit for the r th type ($r=1,2,\dots,s$). The efficiency of the j th decision unit is defined as the ratio of input values to output values, which is modeled as:

$$h_j = \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}}, \text{ And } h_j \leq 1 \quad (1)$$

In the above equation:

$v = (v_1, v_2, \dots, v_m)^T$ is the set of input weights; $\mu = (\mu_1, \mu_2, \dots, \mu_s)^T$ is the set of output weights; $\sum_{i=1}^m v_i x_{ij}$ is the

j th cell composite input value; $\sum_{r=1}^s \mu_r y_{rj}$ is the j th cell composite output value.

The unknowns in the above equation are the weights v and μ in order to maximize the technical efficiency h_j of the j th decision-making unit (DMU), i.e., the other decision-making units are no longer able to exceed this decision-making unit in terms of relative efficiency values. The evaluation model is:

$$\max h_{j0} = \frac{\sum_{r=1}^s \mu_r y_{rj0}}{\sum_{i=1}^m v_i x_{ij0}} \quad (2)$$

$$s.t. \begin{cases} \frac{\sum_{r=1}^s \mu_r y_{rj_0}}{\sum_{i=1}^m v_i x_{ij_0}} \leq 1, & j = 1, 2, \dots, n \\ v = (v_1, v_2, \dots, v_m)^T \geq 0 \\ \mu = (\mu_1, \mu_2, \dots, \mu_s)^T \geq 0 \end{cases} \quad (3)$$

(b) DEA-BCC model

BCC efficiency evaluation model is divided into input-oriented and output-oriented, under the condition of the given output, make the best efforts to reduce the input, this paper uses the input-oriented model in BCC model in the first stage and the third stage.

The two input variable vectors of this paper are set as $p_j(p_{1j}, p_{2j})^T > 0$, $j = 1, 2, \dots, n$, respectively, and the four output variable vectors are set as $q_j(q_{1j}, q_{2j}, q_{3j}, q_{4j})^T > 0$, $j = 1, 2, \dots, n$, where n is 31 provinces. p_{ij} denotes the i th input quantity for the j th province, and q_{rj} denotes the r th output quantity for the j th province. S_i^- denotes the i th input slack. S_r^+ denotes the r th output slack. θ denotes the value of technical efficiency and $0 \leq \theta \leq 1$, the larger θ is, the more effective the decision unit is, and vice versa, the less effective it is.

Input-Oriented Modeling:

$$\min \left[\theta - \varepsilon \left(\sum_{i=1}^m S_i^- + \sum_{r=1}^s S_r^+ \right) \right] \quad (4)$$

$$s.t. \begin{cases} \sum_{j=1}^n \lambda_j p_{ij} + S_i^- = \theta p_{ij_0} \\ \sum_{j=1}^n \lambda_j q_{rj} - S_r^+ = q_{rj_0} \\ \sum_{j=1}^n \lambda_j = 1 \\ S_r^+ \geq 0, S_i^- \geq 0, \lambda_j \geq 0, j = 1, 2, \dots, n \end{cases} \quad (5)$$

b) Stochastic frontier modeling (SFA) in the second stage

In the first step, the target input variables are obtained in the DEA measurement in the first stage, and the value of the slack variable S_{ni} for the n th province of the i th input factor indicator is obtained by calculating the difference between the initial input variables and the target input variables.

In the second step, the slack variables of the input variables are set as the explanatory variables and the environmental variables are set as the explanatory variables, and the stochastic frontier model about the slack variables and the regression variables is established:

$$S_{ni} = f^n(\rho_i; \beta_n) + v_{ni} + \mu_{ni} \quad (6)$$

The $f^n(\rho; \beta_n)$ represents in this paper the relationship between the environmental variables such as fiscal spending and the number of housing companies and the two input slacks, which can be referred to as the environmental values.

In the third step, the log-likelihood function of the stochastic frontier production function is expressed as a function of $\sigma^2 = \sigma_{\mu_n}^2 + \sigma_{v_n}^2$ and $\gamma^2 = \sigma_{\mu_n}^2 / (\sigma_{\mu_n}^2 + \sigma_{v_n}^2)$ instead of $\sigma_{\mu_n}^2$ and $\sigma_{v_n}^2$, $\sigma_{\mu_n}^2$ denotes inefficiency of

management μ_{ni} variance, $\sigma_{v_n}^2$ denotes the variance of the random perturbation v_{ni} , and σ^2 denotes the total variance.

In the fourth step, the management inefficiency term μ_{ni} is separated as well as the randomized perturbation term v_{ni} is calculated:

$$E[v_{ni} | v_{ni} + \mu_{ni}] = S_{ni} - f^n(\rho_i; \beta_n) - E[\mu_{ni} | v_{ni} + \mu_{ni}] \quad (7)$$

Step 5: Adjust the input variables:

$$p_{ni}^* = p_{ni} + [\max(f(\rho_i; \beta_n)) - f(\rho_i; \beta_n)] [\max(v_{ni}) - v_{ni}] \quad (8)$$

p_{ni}^* represents the corrected input values; p_{ni} represents the original input values; $\max(f(\rho_i; \beta_n))$ table is the largest environmental value of all provinces; $\max(v_{ni})$ is the largest stochastic perturbation value of all provinces; and $[\max(f(\rho_i; \beta_n)) - f(\rho_i; \beta_n)]$ is the environment-adjusted value; $[\max(v_{ni}) - v_{ni}]$ denotes the randomized disturbance term adjustment value.

c) Third stage DEA measurement

In order to eliminate the disturbance of the external environment and the influence of random errors, SFA regression was carried out in the second stage, and the original input variables of each province were modified to obtain the modified input variable x_{ni}^* . As in the first stage, the DEA-BCC model was used for the measurement, and only the original inputs were replaced by the modified new input variables, as well as the original output variables.

II. A. 4) Malmquist index method

The Malmquist index [25], also known as Total Factor Productivity (TFP), is widely used to measure production efficiency in sectors such as finance and industry.

The input-output relationship of the decision unit in period t is assumed to be (p_t, q_t) , and that of period $t+1$ is assumed to be (p_{t+1}, q_{t+1}) , and the change in productivity is the change in the adjacent period.

In the technical level of the t period, $M^t(p_{t+1}, q_{t+1}, p_t, q_t) = \frac{D^t(p_{t+1}, q_{t+1})}{D^t(p_t, q_t)}$ denotes the degree of change in the

skill level from the t period to the $t+1$ period, similarly, $M^{t+1}(p_{t+1}, q_{t+1}, p_t, q_t) = \frac{D^{t+1}(p_{t+1}, q_{t+1})}{D^{t+1}(p_t, q_t)}$ indicates the

degree of change in the state of the art from the period of t to the period of $t+1$. Ultimately, the Malmquist index model is obtained by computing the geometric mean of the two Malmquist indices to obtain a composite productivity index from period t to period $t+1$:

$$M^{t,t+1}(p_{t+1}, q_{t+1}, p_t, q_t) = \left[\frac{D^t(p_{t+1}, q_{t+1})}{D^t(p_t, q_t)} * \frac{D^{t+1}(p_{t+1}, q_{t+1})}{D^{t+1}(p_t, q_t)} \right]^{1/2} \quad (9)$$

When $M^{t,t+1} > 1$, it indicates that the level of total factor productivity has increased in the current period compared to the previous period; When $M^{t,t+1} < 1$, it indicates that the level of total factor productivity in the current period has begun to regress and is in a declining phase compared to the previous period; When $M^{t,t+1} = 1$, it means that the level of total factor productivity in the current period remains unchanged compared to the previous period and is in a steady state.

In addition, the Malmquist index is decomposed into two parts: technical efficiency changes and shifts in the efficiency frontier, and the decomposition results are as follows:

$$\begin{aligned}
 M^{t,t+1}(p_{t+1}, q_{t+1}, p_t, q_t) &= \left[\frac{D^t(p_{t+1}, q_{t+1})}{D^t(p_t, q_t)} * \frac{D^{t+1}(p_{t+1}, q_{t+1})}{D^{t+1}(p_t, q_t)} \right]^{1/2} \\
 &= \frac{D^t(p_{t+1}, q_{t+1})}{D^t(p_t, q_t)} * \left[\frac{D^t(p_{t+1}, q_{t+1})}{D^{t+1}(p_{t+1}, q_{t+1})} * \frac{D^t(p_t, q_t)}{D^{t+1}(p_t, q_t)} \right]^{1/2} \\
 &= Effch * Techch
 \end{aligned} \tag{10}$$

II. B. Construction of Malmquist-DEA dynamic evaluation model

Assuming that the number of input and output terms each decision unit (DUM) possesses is denoted by n and m , respectively, and denoting the input and output vectors of the i th DUM by $x_i = (x_{i1}, \dots, x_{in})$ and $y_i = (y_{i1}, \dots, y_{im})$, respectively, $P(x)$ is the set of production possibilities under this constraint. Can denote the distance generated by the decision unit DUM_0 from its production frontier at period t :

$$d_0^t(x_t, y_t) = \inf_{\lambda_t} \{ \lambda_t : (x_t, y_t | \lambda_t) \in P(x_t) \} \tag{11}$$

where, if this DUM wants to bring its output level in line with that of the decision unit on the production frontier by changing the proportions of the output factors, the proportions are denoted as λ_t , $\lambda_t \in [0, 1]$. Then, with constant returns to scale, the TFP growth rate is shown below, using the technology levels T_t and T_{t+1} in periods t and $t+1$, respectively, as benchmarks:

$$M_0^t(x_{t+1}, y_{t+1}, x_t, y_t) = \frac{d_0^t(x_{t+1}, y_{t+1} | C, S)}{d_0^t(x_t, y_t | C, S)} \tag{12}$$

$$M_0^{t+1}(x_{t+1}, y_{t+1}, x_t, y_t) = \frac{d_0^{t+1}(x_{t+1}, y_{t+1} | C, S)}{d_0^{t+1}(x_t, y_t | C, S)} \tag{13}$$

where (x_t, y_t) and (x_{t+1}, y_{t+1}) denote the combination of input and output vectors of DUM_0 in periods t and $t+1$, respectively; and d_0^t and d_0^{t+1} denote the distance function of DUM_0 in periods t and $t+1$, respectively. The Malmquist-DEA index can then be computed from the geometric mean, which reflects the change in total factor productivity over successive periods. To wit:

$$M_0(x_{t+1}, y_{t+1}, x_t, y_t) = \sqrt{M_0^t(x_{t+1}, y_{t+1}, x_t, y_t) \cdot M_0^{t+1}(x_{t+1}, y_{t+1}, x_t, y_t)} \tag{14}$$

If $M_0(x_{t+1}, y_{t+1}, x_t, y_t) > 1$, it means that TFP grows in period t to $t+1$;

If $M_0(x_{t+1}, y_{t+1}, x_t, y_t) < 1$, it means that TFP has decreased from period t to $t+1$.

Moreover, by the intrinsic nature of the Malmquist-DEA index we can transform it to be expressed as the product of the efficiency index (EC) and the technical progress index (TP):

$$\begin{aligned}
 M_0(x_{t+1}, y_{t+1}, x_t, y_t) &= EC \times TP \\
 EC &= \frac{d_0^{t+1}(x_{t+1}, y_{t+1} | C, S)}{d_0^t(x_t, y_t | C, S)} \\
 TP &= \sqrt{\frac{d_0^t(x_{t+1}, y_{t+1} | C, S)}{d_0^{t+1}(x_{t+1}, y_{t+1} | C, S)} \times \frac{d_0^t(x_t, y_t | C, S)}{d_0^{t+1}(x_t, y_t | C, S)}}
 \end{aligned} \tag{15}$$

In particular, the efficiency index (EC) is the ratio of the distance between DUM_0 and the production frontier in the period t to $t+1$, which measures the change in relative efficiency over time. If $EC > 1$, it indicates an improvement in efficiency over the period; conversely, $EC < 1$. The technical progress index (TP), on the other hand, is a measure of the degree of innovation made by the firm in its production technology, with $TP > 1$ and $TP < 1$ measuring the firm's technological progress and regression, respectively.

Similarly, it can be seen that when variable returns to scale are considered, the efficiency index can then be decomposed into a technical efficiency index (PC) and a scale efficiency index (SC), i.e.:

$$\begin{aligned} EC &= PC \times SC \\ PC &= \frac{d_0^{t+1}(x_{t+1}, y_{t+1} | V, S)}{d_0^t(x_t, y_t | V, S)} \\ SC &= \frac{d_0^{t+1}(x_{t+1}, y_{t+1} | C, S) / d_0^{t+1}(x_{t+1}, y_{t+1} | V, S)}{d_0^t(x_t, y_t | C, S) / d_0^t(x_t, y_t | V, S)} \end{aligned} \quad (16)$$

That is, we can derive the expression for the Malmquist-DEA index as:

$$M_0(x_{t+1}, y_{t+1}, x_t, y_t) = PC \times SC \times TP \quad (17)$$

II. C. Differential Analysis Method of Regional Real Estate Markets

II. C. 1) Global spatial autocorrelation

Global spatial autocorrelation is usually measured with the help of the global Moran index. The formula for its calculation is as follows:

$$I = \frac{\sum_{i=1}^n \sum_{j=1}^n w_{ij} (y_i - \bar{y})(y_j - \bar{y})}{s^2 \sum_{i=1}^n \sum_{j=1}^n w_{ij}} \quad (18)$$

where: y_i and y_j are the observed values of the variable for the i th and j th spatial cells; \bar{y} is the mean of the variable for all spatial cells; $s^2 = \frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})^2$, is the variance of the variable across all spatial cells; n is the number of spatial cells; and w_{ij} is the spatial weight after row normalization between the i th and j th spatial cells. The spatial units in this study were each monitoring water station in Zhejiang Province, and the study variables were the water quality composite index and dissolved oxygen content of each monitoring water station.

The original hypothesis of the global Moran test is that the spatial autocorrelation of the study variables is not significant, and the constructed Z-test statistic is specified as follows:

$$Z_I = \frac{I - E[I]}{\sqrt{Var[I]}} \quad (19)$$

where: $E[I] = \frac{-1}{(n-1)}$, is the mean of I , and $Var[I] = E[I^2] - E[I]^2$, is the variance of I . The rejection domain of this test is $|Z_I| \geq 1.96$ under the condition that the significance level is taken to be 0.1, and if the value of the test statistic of Z is within the rejection domain, the original hypothesis is rejected and the variables are considered to have significant spatial autocorrelation [26].

Moran's index is a rational number with a value domain of $[-1, 1]$ and the magnitude of its value measures the strength of spatial autocorrelation. When $Moran's I > 0$, it means the spatial positive correlation among the spatial units, and the larger its value, the more obvious spatial aggregation effect; when $Moran's I < 0$, it means the spatial negative correlation among the spatial units, and the smaller its value, the larger the spatial difference; and when $Moran's I = 0$, it means spatial stochasticity among the spatial units.

II. C. 2) Local spatial autocorrelation

Local spatial autocorrelation discrimination can be carried out by local spatial Moran's index and hotspot analysis.

The local spatial Moran index studies the local spatial autocorrelation of variables, mainly using the attribute values of the spatial cell itself to compare with the surrounding points, which can find the hidden outliers. For the i th spatial cell, its local Moran's index calculation formula is shown below:

$$I_i = \frac{y_i - \bar{y}}{s_i^2} \sum_{j=1, j \neq i}^n w_{ij} (y_j - \bar{y}) \quad (20)$$

where y_j is the value of the variable for the remaining j th cell in space; \bar{y} is the mean value of the variable for all space cells; n is the total number of space cells; $s_i^2 = \frac{\sum_{j=1, j \neq i}^n (y_j - \bar{y})^2}{n-1}$, the variance of the variable for all cells except the i th cell; w_{ij} is the value of the spatial weight between the i th and j th spatial cells.

A positive value of the spatial local Moran index indicates that the spatial cell has neighboring cells containing equally high or low values of the variable and there is spatial clustering; a negative value of the local Moran index indicates that the spatial cell has neighboring cells containing different values and the cell is an outlier.

The local Moran test is used to test whether the spatial clustering is statistically significant, with the original hypothesis that the spatial clustering around the spatial unit is not significant, and its test statistic is:

$$Z_{I_i} = \frac{I_i - E[I_i]}{\sqrt{Var[I_i]}} \quad (21)$$

where: $E[I_i] = -\frac{\sum_{j=1, j \neq i}^n w_{ij}}{n-1}$, is the mean of I_i ; $Var[I_i] = E[I_i^2] - E[I_i]^2$, is the variance of I_i ; and at the level of significance of 0.05, the corresponding rejection domain is $|Z_{I_i}| \geq 1.96$.

The clustering type of the research unit can be determined by calculating the I_i value and the Z_{I_i} value. When $I_i > 0$ and $Z_{I_i} \geq 1.96$, it indicates that there is a high-high clustering with high values enclosing high values (HH); when $I_i < 0$ and $Z_{I_i} \leq -1.96$, it indicates that there is a low-low clustering with low values enclosing low values (LL); when $I_i < 0$ and $Z_{I_i} \leq -1.96$, it indicates that there is a low-low clustering with low values enclosing low-values (LL); when $I_i > 0$ and $Z_{I_i} \leq -1.96$, it indicates the existence of a low-high clustering (LH) with low values surrounding high values; when $I_i < 0$ and $Z_{I_i} \geq 1.96$, it indicates the existence of a high-low clustering (HL) with high values surrounding low values; other than that, the spatial distribution of this metric is considered to be randomly distributed.

III. Analysis of differences and convergence in China's regional real estate markets

III. A. Empirical analysis of the efficiency of China's real estate industry

In this paper, the relevant input-output indicators of the real estate industry for each year from 2017 to 2024 were processed using DEAP software to measure the technical efficiency, pure technical efficiency and scale efficiency and total factor productivity of 31 provinces, municipalities and autonomous regions in China, respectively.

III. A. 1) Analysis of DEA measurement results

Using the DEA method, the real estate industry efficiency of China's real estate industry in 31 provinces was analyzed in a cross-sectional comparison. The DEAP software was used to process and derive the technical efficiency, pure technical efficiency and scale efficiency values, as well as the returns to scale of China's overall real estate industry for each year from 2017 to 2024, and the results of the DEA analysis are collated as shown in Table 2. It can be seen that, from the technical efficiency development trend, from 2017 to 2019, the technical efficiency of China's real estate industry showed a downward trend, 2020 was a clear turning point, with a small increase, and then declined in 2021, after which the comprehensive technical efficiency steadily increased until 2024. Decomposing technical efficiency into perspective, the level of pure technical efficiency of China's real estate industry has not been high, the decline of pure technical efficiency is the main reason for the decline of technical efficiency in China's real estate industry, and pure technical inefficiency is the main reason for the lower technical efficiency of China's real estate industry. Although the scale efficiency of each year has not reached the optimum, its value is only less than 1, with little fluctuation, and the scale efficiency value of each year is greater than the pure technical efficiency value, indicating that improving the technical efficiency of the real estate industry is the key to improving the input-output efficiency of China's real estate industry, and it is necessary to improve the output capacity of China's real estate industry to fully utilize the current resources.

In terms of returns to scale, from 2017 to 2019, the number of provinces with incremental returns to scale is the largest and increasing, reaching 24 by 2019. From 2020 onwards, the number of provinces with decreasing returns to scale begins to predominate, while the number of provinces with constant returns to scale in 2023 and 2024 predominates. According to economics, when in a state of increasing returns to scale, this indicates that greater

outputs can be achieved by increasing inputs. When the returns to scale are decreasing, it means that there is no possibility of increasing inputs to achieve greater outputs. The state of constant returns to scale is an ideal state, indicating that the inputs and outputs have already reached the optimum, which shows that the input and output conditions of China's real estate industry have already matured.

Table 2: DEA analysis of the results

Year	Efficiency mean			Scale benefit		
	Crste	Vrste	Scale	Increasing	Invariant	Diminishing
2017	0.7154	0.7453	0.9671	12	9	10
2018	0.6684	0.6989	0.9793	21	6	4
2019	0.6003	0.6395	0.9606	24	5	2
2020	0.6766	0.7204	0.9417	5	5	21
2021	0.6356	0.6559	0.9672	16	5	10
2022	0.7752	0.7954	0.9606	4	8	19
2023	0.7831	0.8071	0.9489	3	14	14
2024	0.8112	0.8484	0.9677	2	17	12

III. A. 2) Analysis of DEA Measurement Results of Real Estate Industry by Province

Taking the DEA measurement results of China's real estate industry in each province in 2024 as an example to be analyzed, from the perspective of the efficiency value of each province and the input redundancy of the non-efficient decision-making unit, the results of the DEA efficiency measurement of China's 31 provinces in 2024 are shown in Table 3. The average technical efficiency, the pure technical efficiency and the scale efficiency of China's real estate industry in 2024 are not optimal. Among them, the technical efficiency is 0.81524, indicating that there is 18.476% waste of resources in China's real estate industry; the pure technical efficiency is 0.84511, indicating that the technical level of China's real estate industry is still to be improved; the scale efficiency is the highest at 0.96368, indicating that the scale of China's real estate industry's investment has been expanded, and the industry's economy of scale has been basically realized.

From the perspective of the three major regions in the east, west and east, the technical efficiency, pure technical efficiency and scale efficiency of the real estate industry in the eastern region has a clear absolute advantage, mainly because of the overall economic strength of the eastern region, whether it is the infrastructure, manpower investment or technical level, management level is much larger than the central and western regions, from the input indicators, the value of various input and output indicators is almost equal to the sum of the central and western regions, and some of them are even much larger than the sum of the central and western regions, and its advantage is the same as that of the central and western regions. Some of them are even much larger than the sum of the central and western regions, so there is no doubt about its superiority. While the central region, although the level of economic development is better than the western region, the technical efficiency of its real estate industry fails to reflect its advantages, and even its technical efficiency is slightly lower than that of the western region, as can be seen from the results of the analysis of the accompanying table, the central provinces, although the input indexes in the amount of absolute advantage, but the redundancy is high, which is a direct response to the irrationality of the inputs of the real estate industry in central China, and the need to adjust the structure of its inputs and outputs.

III. A. 3) Results of Malmquist Index Measurement for the National Real Estate Industry

The DEA analysis method can only make horizontal comparisons of decision-making units at the same point in time, and the Malmquist index can analyze the vertical development trend for each decision-making unit, and the analysis results are as follows.

The annual average Malmquist index of the overall efficiency of China's real estate industry during the period 2017-2024 is shown in Table 4. From the data in the table, it can be seen that the overall total factor productivity (TFP) index of China's real estate industry from 2017 to 2024 is 1.0277 increased by 2.77%, of which the technical efficiency increased by an average of 3.05%, while the index of technological progress declined by 0.27%, so that the productivity growth of China's real estate industry comes from the improvement of technical efficiency, which is mainly the pure technical efficiency, and the scale efficiency has a Before 2020, the total factor productivity index of the real estate industry is greater than 1, and the real estate industry realizes steady growth, and the TFP value in 2020 has a significant drop to the lowest value of 0.7576, which is mainly due to the impact of the global economic crisis caused by the U.S. subprime mortgage crisis on China's real estate market. In 2021, the total factor productivity has a significant increase to reach the maximum value of 1.4171, and the technical progress index decreases by 0.27%. 1.4171, and the technological progress index also reaches the maximum value of 1.3782, mainly due to the government's 4 trillion investment plan in 2020, which massively promotes the construction of infrastructure such

as guaranteed housing projects and largely stimulates the moderate development of China's real estate industry. After 2021, constrained by the technological progress index, the TFP index of the real estate industry slides down again and is slightly less than 1, which indicates that China's real estate industry is slightly deficient in technological innovation and the level of introducing new technologies.

Table 3: In 2024, the DEA in the provinces calculated the results

Province	Crste	Vrste	Scale	Scale benefit
Beijing	1	1	1	-
Tianjin	1	1	1	-
Hebei	0.6887	0.753	0.9057	drs
Liaoning	1	1	1	-
Shanghai	1	1	1	-
Jiangsu	0.9646	1	0.9581	drs
Zhejiang	0.9253	0.9716	0.9523	drs
Fujian	0.924	0.9464	0.9718	drs
Shandong	0.7386	0.727	1	-
Guangdong	1	1	1	-
Hainan	0.9538	0.9962	0.9598	drs
Shanxi	0.4981	0.5043	1	-
Jilin	0.6914	0.7027	0.9961	irs
Heilongjiang	0.9775	1.0114	0.9733	drs
Anhui	0.6559	0.6721	0.9898	drs
Jiangxi	1	1	1	-
Henan	0.6844	0.8132	0.8518	drs
Hubei	0.716	0.7228	0.9822	drs
Hunan	0.8207	0.8985	0.9142	drs
Neimenggu	0.7081	0.7108	1	-
Guangxi	0.6348	0.6242	1	-
Chongqing	0.785	0.7799	0.9915	irs
Sichuan	0.9204	0.9137	1	-
Guizhou	0.6855	0.6801	1	-
Yunnan	0.6555	0.6438	0.998	irs
Xizang	1	1	1	-
Shanxi	0.8626	0.9829	0.8661	drs
Gansu	0.5692	0.578	0.9738	drs
Qinghai	0.5183	0.87	0.5895	drs
Ningxia	0.6941	0.6958	1	-
Xinjiang	1	1	1	-
East	0.92682	0.94493	0.97706	
Middle	0.7555	0.79063	0.96343	
West	0.75279	0.78993	0.95158	
Mean	0.81524	0.84511	0.96368	

In order to analyze the trend of total factor productivity more intuitively, the annual average change of Malmquist index and its decomposition in China's real estate industry during the period of 2017-2024 is shown in Figure 1. By observing the trend of the graph lines, it can be found that the folding line of total factor productivity and the folding line of the technical change index have the same trend of change, indicating that the technical change index has the greatest impact on total factor productivity, that is, the change of productivity in China's real estate industry is mainly driven by technological innovation or the introduction of new technologies. The trend of the technical efficiency index and the line of the pure technical efficiency trend almost coincide, indicating that the pure technical efficiency is the main driver of the technical efficiency index, and the line of the scale efficiency index has the smallest fluctuation, and the change is relatively smooth.

Table 4: The overall efficiency of the real estate industry is the Moran's I index

Year	Total factor productivity (tfpch)	Technical efficiency index (effch)	Technological progress index (techch)	Pure technological progress index (pech)	Scale efficiency index (sech)
2017-2018	1.0020	0.9367	1.0752	0.9306	1.0064
2018-2019	1.1138	0.9093	1.2190	0.9174	0.9778
2019-2020	0.7576	1.1327	0.6613	1.1545	0.9850
2020-2021	1.4171	0.9080	1.3782	0.8675	1.0301
2021-2022	0.9946	1.2553	0.7923	1.2603	0.9888
2022-2023	0.9731	1.0248	0.9422	1.0307	0.9939
2023-2024	0.9359	1.0469	0.9129	1.0445	1.0125
Mean	1.0277	1.0305	0.9973	1.0294	0.9992

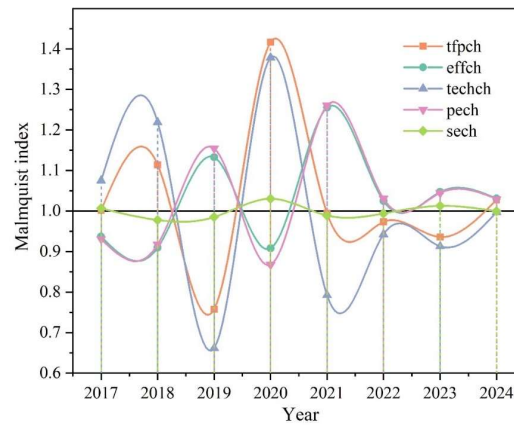


Figure 1: Changes in the annual Malmquist index in China's real estate industry

III. B. Analysis of spatial agglomeration and evolutionary trends in real estate development

III. B. 1) Real estate development global autocorrelation analysis

In this paper, with the help of the spatial analysis module of Geoda software, the global Moran's index of the Yangtze River Economic Belt for 2010-2024 is calculated, so as to explore the global spatial autocorrelation of real estate development in the Yangtze River Economic Belt, and the global Moran's I of the Yangtze River Economic Belt for real estate development is shown in Table 5.

From the global Moran's I value, the global Moran's I of real estate development continues to go down, indicating that the spatial autocorrelation of regional real estate development is gradually weakening, which may be due to the fact that there are large differences in real estate development of the provinces in the early period, and China's real estate development has an obvious time sequential feature, with the real estate development of the developed provinces in the early period being larger in volume, and that of the neighboring underdeveloped provinces being comparatively smaller in scale. The spatial autocorrelation degree of regional real estate development is low because the real estate development volume of developed provinces is large while the real estate development scale of neighboring underdeveloped provinces is relatively small, and therefore the set circulation effect of real estate development factors between provinces is insufficient.

III. B. 2) Characterization of trends in the evolution of real estate development

(1) Traditional Markov chain model construction

The traditional Markov chain model is based on the type transfer process of the whole real estate development, constructing a Markov chain probability transfer matrix of $N \times N$ order, with probability value P_{ij} , so as to represent the dynamic evolution characteristics of real estate development. Presuming that P_{ij} is the probability of real estate development in a province to transfer from the type E_i in the t year to the type E_j in the $t+1$ year, the transfer probability of real estate development can be represented by the transfer frequency of the transfer probability, the specific expression is as follows:

$$P_{ij}(E_i \rightarrow E_j) = \frac{n_{ij}}{n_i} \quad (22)$$

where n_{ij} represents the total number of evaluation units in the time domain of the study in which the real estate development is transferred from type E_i to type E_j when the i level is transferred to the j level; and n_i represents the number of provinces in which E_i is at the i level. In addition, according to the increasing, constant, and decreasing level of real estate development of evaluation units to define the transfer direction, this paper divides the real estate development of the Yangtze River Economic Zone into four types based on the quartile division criteria with 25%, 50%, and 75% as the boundaries. The traditional Markov chain probability transfer matrix is shown in Table 6.

Table 5: Real estate development ecological efficiency global

Moran's I Year	Global Moran's I	P value	Z value
2010	0.2364***	0.0001	3.2955
2011	0.1375**	0.0161	2.2447
2012	0.1973***	0.0004	2.9383
2013	0.2272***	0.0001	3.2644
2014	0.2047***	0.0001	2.804
2015	0.1392**	0.0249	2.0678
2016	0.1328**	0.0255	1.9715
2017	0.1277**	0.0268	2.0685
2018	0.0969*	0.0629	1.5202
2019	0.0891*	0.0871	1.341
2020	0.1098**	0.0425	1.7484
2021	0.0817*	0.0921	1.2545
2022	0.1320**	0.0251	2.0157
2023	0.1087**	0.0479	1.6479
2024	0.2452***	0.0001	3.3833

Note: ***, ** and * represent 1%, 5% and 10% significance levels, respectively.

Table 6: Traditional markov chain probability transfer matrix

ti/ti+1	Type I	Type II	Type III	Type IV
Type I	P11	P12	P13	P14
Type II	P21	P22	P23	P24
Type III	P31	P32	P33	P34
Type IV	P41	P42	P43	P44

In this paper, with the help of traditional Markov chain and based on the quartile division standard, the real estate development in the Yangtze River Economic Zone in 2010-2024 is divided into Type I, Type II, Type III and Type IV, and the larger value of the type indicates the higher value of the real estate development, and the results of the probability transfer matrix of the traditional Markov chain are shown in Table 7.

In terms of maintaining the original type, from the probability transfer matrix of real estate development, it can be seen that the probabilities on the main diagonal of the four types are much higher than the probabilities on the non-main diagonal, of which P_{IV-IV} (0.7588) is the maximum transfer probability value of the main diagonal, indicating that the probability of the real estate development will be stable to maintain the original type in the future evolution process.

In terms of extreme convergence, the probability of the two ends of the real estate development diagonal is higher than the middle, with probability values of P_{II-IV} (0.7588) > P_{I-I} (0.7566) > P_{II-II} (0.6217) > $P_{III-III}$ (0.5462), which indicates that the real estate development exhibits a tendency to cluster to higher (lower) levels, i.e., there is a convergence phenomenon.

0.1807, and the minimum probability is 0.0202, which is much lower than the probability on the diagonal line, indicating that it is more difficult for real estate development to realize inter-stage transfer in a short period of time.

In terms of risk prediction, the probability of real estate development transferring to a low level type is relatively low, the probability of Type II transferring to Type I is lower than its probability of transferring to Type III (P_{II-I} (0.115) < P_{II-III} (0.1807)) and the probability of transferring from Type III to Type II is also lower than its

probability of transferring to Type IV transfer probability ($P_{III-II}(0.1664) < P_{III-IV}(0.1732)$), indicating that real estate development tends to transfer to a higher level, with a lower degree of risk of a decrease in level.

Table 7: The probability transfer matrix of the traditional markov chain

Local Type	Type I	Type II	Type III	Type IV
	<25%	25%-50%	50%-75%	>75%
Type I	0.7566	0.1139	0.1075	0.0202
Type II	0.115	0.6217	0.1807	0.0706
Type III	0.1072	0.1664	0.5462	0.1732
Type IV	0.0344	0.0694	0.144	0.7588

(2) Spatial Markov chain model construction

Spatial Markov chain is based on the traditional Markov chain, the introduction of spatial lag in the Markov chain probability transfer matrix, so as to explore the influence of the same attribute value of neighboring things in geospatial, the specific expression is as follows:

$$Lag = Y_i W_{ij} \quad (23)$$

Lag represents the spatial lag value of evaluation unit i , Y_i represents the attribute value of evaluation unit i , and W_{ij} represents the relationship matrix between the evaluation unit and the neighboring regions, i.e., the spatial weight matrix is constructed based on the center-of-mass distance of the evaluation unit.

Table 8: Markov chain probability transfer matrix for real estate development

Space lag Type	Local Type	Type I	Type II	Type III	Type IV
		<25%	25%-50%	50%-75%	>75%
Type I	Type I	0.7794	0.0844	0.1148	0.0214
	Type II	0.1341	0.6119	0.1770	0.0770
	Type III	0.1321	0.1610	0.5643	0.1426
	Type IV	0.0380	0.1205	0.1469	0.6946
Type II	Type I	0.7463	0.1609	0.0600	0.0328
	Type II	0.1273	0.6471	0.1613	0.0643
	Type III	0.1460	0.1697	0.5270	0.1633
	Type IV	0.0571	0.0887	0.1702	0.6840
Type III	Type I	0.7306	0.1252	0.1290	0.0152
	Type II	0.1163	0.6130	0.2011	0.0707
	Type III	0.0882	0.1837	0.5362	0.1919
	Type IV	0.0328	0.0482	0.0942	0.8248
Type IV	Type I	0.6978	0.1190	0.1400	0.0432
	Type II	0.1082	0.6371	0.1840	0.0707
	Type III	0.0874	0.1638	0.5558	0.1930
	Type IV	0.0247	0.0368	0.1669	0.7716

The spatial Markov chain probability transfer matrix of real estate development in the Yangtze River Economic Zone is shown in Table 8. By comparing the results of the traditional Markov chain probability transfer matrix, it can be concluded that the regional real estate development has the following spatial evolution characteristics:

The spatial agglomeration effect significantly affects the dynamic evolution process of real estate development. Under the condition of the difference in the efficiency value of real estate development in neighboring areas, the probability of transferring the type of regional real estate development is completely different from the traditional Markov chain transfer probability. In the spatial Markov chain probability transfer matrix, when a province is adjacent to the provinces of type I and type II, the probability of real estate development transferring from type III to type IV is $P_{III-IV(I)}(0.1426)$, $P_{III-IV(II)}(0.1633)$, all of which are lower than the transfer probability measured by the traditional Markov chain. If a province is adjacent to a province of type III and type IV, the probability of real estate development transferring from type III to type IV is $P_{III-IV(III)}(0.1919)$, $P_{III-IV(IV)}(0.1930)$, which are all higher than

the transfer probabilities measured by traditional Markov chains, i.e., the spatial agglomeration effect has a significant assimilation effect on the dynamic evolution of real estate development.

The spatial agglomeration effect has an important impact on the dynamic transfer process of real estate development. When the neighboring province has a high level of real estate development, the probability of real estate development transferring to a high level type in its own province increases, while the degree of risk of real estate development transferring to a low level type is greater when it is adjacent to a province with a low level of real estate development. For example, in the context of neighborhoods with high real estate development, $P_{I-II(II)}(0.1609) > P_{I-II}(0.1139)$, $P_{II-III(III)}(0.2011) > P_{II-III}(0.1807)$, while in the context of areas with low real estate development, $P_{II-I(I)}(0.1341) > P_{II-I}(0.115)$, $P_{III-II(II)}(0.1697) > P_{III-II}(0.1664)$. Therefore, provinces with low levels of real estate development should fully absorb the spatial spillovers from high-level provinces, thus promoting the leapfrog improvement of local levels.

IV. Conclusion

Starting from the differences and convergence of regional real estate markets, this paper constructs a dynamic evaluation model based on Malmquist-DEA to reflect the changes of total factor productivity in consecutive periods, and adopts the Moran index and spatial autocorrelation model to analyze the spatial agglomeration and evolutionary characteristics of regional real estate development, so as to provide theoretical support for the improvement of the development efficiency of China's real estate industry.

(1) The technical efficiency, pure technical efficiency and scale efficiency of China's real estate industry were analyzed in a horizontal comparison using the DEA method. It is concluded that the scale efficiency value of China's real estate industry in all the years is higher, and the pure technical inefficiency is the main reason for the lower technical efficiency of China's real estate industry; the efficiency of the real estate industry shows obvious regional differences.

(2) The total factor productivity of China's real estate industry was analyzed in terms of its vertical development status by using the Malmquist index. It is obtained that the overall TFP index of China's real estate industry from 2017 to 2024 is 1.0277 improved by 2.77%, in which the technical efficiency is improved by 3.05% on average, while the technical progress index is decreased by 0.27%. Meanwhile, the total factor productivity of China's real estate industry also shows strong regional variability.

(3) From the viewpoint of the evolutionary characteristics of real estate development, the real estate development in the Yangtze River Economic Zone has convergence characteristics. From the traditional Markov chain, real estate development is more inclined to maintain the original type, and it is difficult to realize the transfer of cross-stage level in the short term, and the degree of risk of transferring to a low level state is low. In addition, the spatial agglomeration effect has a significant impact on the evolution of real estate development, and the transformation of cities will be influenced by neighboring cities, and the direction of influence is related to the level of neighboring cities.

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