International Journal for Housing Science and Its Applications
Publish August 6, 2025. Volume 46, Issue 3 Pages 6722-6733

https://doi.org/10.70517/ijhsa463577

Strategies for Piano Practice Time Allocation and Effectiveness Enhancement Based on Data Envelopment Analysis

Jing Fan[1,*]
[1] College of Music and Dance, Fuyang Normal University, Fuyang, Anhui, 236000, China
Corresponding authors: (e-mail: fanny9047@163.com).

Abstract Piano learning effect is closely related to the allocation of practice time, but most piano learners lack scientific practice time management strategies. Based on the data envelopment analysis method, this study investigates the evaluation of piano practice time allocation and the strategy of effect improvement. Piano learners' practice data were obtained through a questionnaire survey, quantitatively evaluated using the interval super-efficiency DEA model, and the learners' practice habit characteristics were analyzed. The results of the study show that among the 21 learners sampled, only 19.05% of the learners' piano practice time allocation is interval DEA effective, 52.38% is interval DEA partially effective, and 28.57% is interval DEA ineffective, indicating that there is room for optimization of piano practice time allocation for most of the learners. It was found that 34.26% of the learners could insist on practicing the piano every day, and 45.53% of the learners maintained an average daily practice time of about 1 hour. For the practice habit, only 25.98% of the learners would practice hand muscles and techniques, and 20.59% of the learners had the habit of making practice notes. Based on the evaluation results, this study proposes strategies to improve the effectiveness of piano practice: strengthening fingering practice and reflection, scientifically controlling practice time, focusing on technical difficulties, and actively learning from others' experiences. These strategies are of great significance in optimizing piano learners' practice methods and improving practice efficiency.

Index Terms Piano practice, Time allocation, Data envelopment analysis, Interval super-efficiency, Effectiveness enhancement, Practice habits

I. Introduction

Piano playing is an extremely skillful art, and grueling repetitive practice is bound to be an extremely important part of the learning process for piano learners. The learning of piano skills requires the accumulation of time. It is a rather difficult task to enable students to master certain playing skills and possess a certain playing ability within a limited time. In this process of interaction between "teaching" and "learning", the quality of students' practice is particularly important. Good practice results will make teaching twice as effective with half the effort [1]-[3]. Without the guidance of correct theories and methods, they are just doing repetitive mechanical movements, instead, they will consolidate the mistakes and even form some bad habits that are difficult to adjust [4]-[6]. And because students have to be busy with many specialized courses, the exclusive time for piano practice is reduced. Reasonable utilization of piano practice time can greatly improve the efficiency of practice [7]. Appropriate allocation of practice time and decentralization of practice requirements in piano practice are more efficient than “vigorous practice” that concentrates too much on practice time and practice requirements [8]-[10]. How to make students in the limited time, rational allocation of practice time, improve the effect of practice, how to use scientific and effective methods of practice, as far as possible to achieve the purpose of teaching, has become a problem that has to be studied.

In today's management field, in order to effectively assess and optimize the efficiency of various production, service or decision-making processes, some powerful analytical tools are needed, and data envelopment analysis is one of them [11]. Data Envelopment Analysis is a method used to assess the relative efficiency of multiple decision-making units. These decision units can be businesses, departments, hospitals, schools, etc [12]. It is an effective way to manage time efficiency by comparing the inputs and outputs of these units to determine which units are able to maximize outputs for a given set of resources or minimize inputs for a given level of outputs [13].

The art of piano, as a superb form of performing art, requires learners to invest a great deal of time and energy in systematic training. The practicing effect of piano learners is not only related to the total length of practicing, but also closely related to the reasonable distribution of practicing time and scientific practicing methods. For a long time, the research on practice efficiency in the field of piano education mainly focuses on qualitative analysis and experience summarization, and lacks an objective quantitative evaluation system. Although many piano learners have invested a lot of time in practicing, due to the lack of scientific practicing strategies and time management methods, the effect of practicing is often unsatisfactory. Especially for amateur piano learners, due to the time constraints of their occupation or studies, how to achieve the best learning effect within the limited practicing time has become an urgent problem to be solved. Scholars at home and abroad have conducted extensive research on piano practice methods, but most of the research focuses on technique training, repertoire analysis, etc., and there are relatively few scientific studies on the allocation of practice time. Existing studies mostly rely on qualitative description and subjective experience, which makes it difficult to provide learners with personalized and precise practice guidance. As a non-parametric efficiency evaluation method based on multiple inputs and multiple outputs, data envelopment analysis is characterized by strong objectivity and wide applicability, and it is of great significance to apply it to the evaluation of piano practice time allocation. By establishing a quantitative evaluation model of piano practice time allocation, this study can provide piano learners with scientific practical guidance to help them optimize practice strategies and improve practice efficiency. First, we obtained the piano learners' practice data through a questionnaire survey, including multiple dimensions such as practice mode, practice frequency, practice time, and practice habits. Then, the interval DEA method was used to conduct a preliminary evaluation of piano practice time allocation, and it was found that the traditional interval DEA model has limitations in distinguishing effective decision-making units. To overcome this defect, the interval super-efficiency DEA model is introduced to realize the sufficient ordering of decision-making units by establishing the likelihood matrix, so as to evaluate the efficiency of piano practice time allocation more comprehensively. Finally, based on the evaluation results, the problems of piano practice are analyzed, and strategies to enhance the effectiveness of piano practice are proposed, such as strengthening fingering practice, controlling the practice time, focusing on the technical difficulties, and learning from others' experiences, which provide practical guidance suggestions for piano learners.

II. Piano practice survey and analysis

II. A. Subjects of the survey

The data for this study were collected from five social art training organizations in a city. The questionnaire survey was mainly conducted on piano learners, 250 questionnaires were distributed and 232 valid questionnaires were retrieved, with an effective recovery rate of 92.8%.

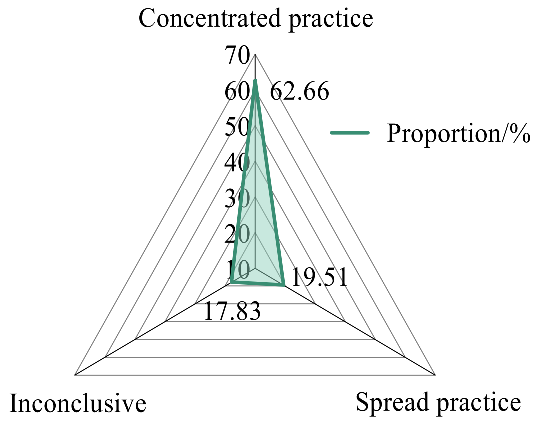
II. B. Questionnaire design

The questionnaire was administered to the students and was divided into 3 dimensions, including students' basic information, time control of practicing piano and habits of practicing piano. Among them, the time control of practicing the piano included 3 dimensions such as practicing mode X1 (centralized or decentralized practice), practicing frequency X2, and total time of practicing X3. Practicing habits include whether or not to do hand muscle exercises and technique exercises before practicing X4, whether or not to have the habit of reading music and sight-reading before practicing X5, whether or not to do some basic music theory, general knowledge of music, and music appreciation X6, whether or not to learn the comprehensive knowledge of a piece before practicing a new piece X7, whether or not to listen to versions of all kinds of famous artists' performances of the piece studied X8, whether or not to spend more time practicing a piece you like, but spend less time practicing a piece you aren't interested in, and whether or not to spend more time practicing a piece you don't like, and spend more time practicing a piece you don't want. X9, whether they have the habit of making practice notes during practice X10, whether they usually record and videotape their practice X11, whether they fulfill the teacher's requirements X12, and other 9 dimensions. The collected data of practice time control can be used as the sample data for the later study of piano practice time allocation.

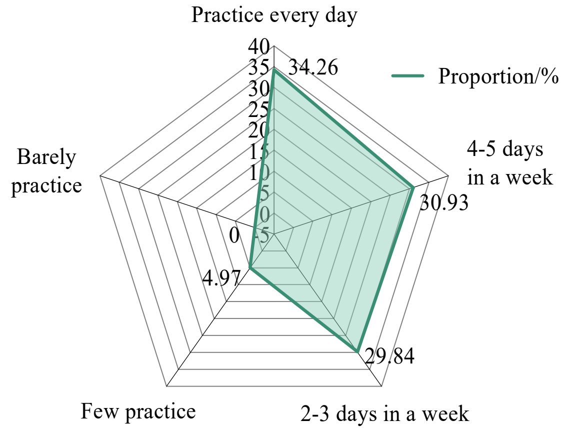
II. C. Analysis of findings

II. C. 1) Exercise time control

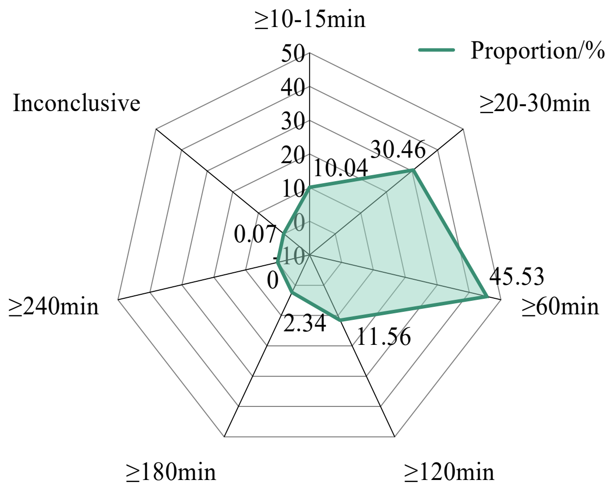
According to the survey data, the results of learners' piano practice time control are shown in Figure 1, (a) ~ (c) represent the findings of the practice methods, practice frequency and practice time respectively. Most of the students used centralized practice method to practice the piano, which accounted for 62.66%, another part used decentralized practice, and a part was uncertain. Figure (b) shows that 34.26% of the students are able to practice every day, 30.93% practice 4-5 days in a week, and another 34.81% rarely practice. In figure (c) 40% of the students practiced for less than half an hour, 45.53% practiced for about one hour, and only 13.97% of the students practiced for more than two hours.



(a)Piano practice way



(b)Piano practice frequency



(c)Piano practice time

Figure 1: The results of the learners' piano practice time control

II. C. 2) Analysis of practice habits

Figure 2 shows the survey data of learners' piano practicing habits. The proportion of learners who have the habit of reading music and sight-singing before practicing X5, who have the habit of practicing more of the pieces they like but are not interested in X9, who usually record and video record their own practice X11, and who can fulfill the teacher's requirements X12 are all above 50%. 25.98% of the learners will do hand muscle practice and technique practice before each practice session, 31.82% of the learners will do some basic music theory, music general knowledge and 31.82% of the learners will do some basic music theory, general knowledge of music and music appreciation, only 26.37% of the learners will learn the comprehensive knowledge of the piece before practicing the new piece, 36.61% of the learners will listen to the versions of the pieces they learnt played by various famous artists and analyze the differences and styles of the various versions, and the learners who have the habit of taking notes when practicing accounted for only 20.59%.

Through the above survey, it is found that the sample piano learners still have the problems of lack of time management of practicing, lack of good learning atmosphere and practicing habits.

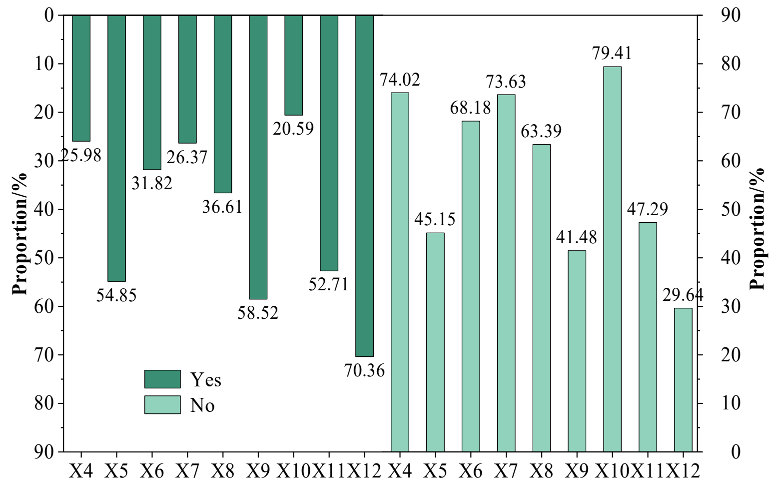


Figure 2: The data of the students' piano practice habits

III. DEA-based evaluation of piano practice time allocation

III. A. Research methodology

III. A. 1) DEA methodology

Data Envelopment Analysis (DEA) is a non-parametric efficiency analysis and evaluation method based on relative comparisons between the evaluated decision units. This method uses a mathematical planning model to measure the corresponding production frontiers through a set of observations of input and output indicators, and the relative efficiency can be obtained by comparing each decision-making unit with the effective production frontiers. The "production frontier" here is essentially an envelope of better performing DMUs (effective DMUs) as judged by the input-output indicators of DMUs. It can be seen that DEA can not only make full use of the existing information in the evaluation process, but also does not need to set the production relationship from outside, and has strong objectivity.

(1) CCR model based on constant returns to scale

The CCR model is based on the assumption of constant returns to scale (CRS), and the CCR model consists of two main planning styles, the input-oriented planning style and the output-oriented planning style. Input-oriented CCR model is to maximize the output-input ratio and output-oriented CCR model is to minimize the input-output ratio.

Suppose there are n DMUs that use m inputs to produce q outputs. Each DMU is denoted as $DMU_j(j=1,2,\cdots,n)$, and observable inputs and outputs are denoted as $x_j=(x_{1j},x_{2j},\cdots,x_{mj})^T$ and $y_j=(y_{1j},y_{2j},\cdots y_{qj})^T$. The $x_j\geq 0, y_j\geq 0$ specifies that neither the input indicator data nor the output indicator data is less than zero, and at least one of the indicator data is greater than zero. Then the input-oriented CCR model is:

$$\begin{aligned} \max \quad & \sum_{r=1}^{q}\mu_r y_{rk} \\ s.t. \quad & \sum_{r=1}^{q}\mu_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}\leq 0 \\ & \sum_{i=1}^{r=1}v_i x_{ik}=1 \\ & v_i\geq 0;\mu_r\geq 0 \\ & i=1,2,...,m; r=1,2,...,q; j=1,2,...,n. \end{aligned} \tag{1}$$

The optimal solution of DMU_k out of the efficiency value can be calculated by model (1) as θ_k^* with $0<\theta_k^*\leq 1$, where $\theta_k^*=1$ is said to be the DEA valid for DMU_k and $\theta_k^*<1$, then DMU_k is said to be DEA ineffective.

Similarly the output-oriented CCR model is:

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} v_i x_{ik} \\ s.t. \quad & \sum_{r=1}^{q} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \\ & \sum_{r=1} \mu_r y_{rk} = 1 \\ & v_i \ge 0; \mu_r \ge 0 \\ & i = 1,2,...,m; r = 1,2,...,q; j = 1,2,...,n. \end{aligned} \tag{2}$$

The optimal solution of DMU_k out of the efficiency value can be calculated by model (2) as φ_k^*, $\varphi_k^* \ge 1$, when $\varphi_k^* = 1$, it is said that DMU_k is the DEA valid, otherwise it is called DMU_k is DEA invalid. It is generally stipulated that the inverse of φ is adopted as the efficiency value of DMU_k, and for the same DMU_k, $\theta_k^* = 1/\varphi_k^*$, i.e., the efficiency measure is the same regardless of whether it is an input-oriented CCR model or an output-oriented CCR model.

(2) BCC model based on variable returns to scale

The basic assumption of the CCR model is that the return on scale remains unchanged. The BCC model is based on the dual model of the CCR model and takes into account the constraint condition $\Sigma_{j=1}^{n} \lambda_j = 1$. This enables the production scale of the evaluated DMU to be consistent with that of the projection points.

Model (3) is the input-oriented BCC model:

$$\begin{aligned} \max \quad & \sum_{r=1}^{q} \mu_r y_{rk} - \mu_0 \\ s.t. \quad & \sum_{r=1}^{q} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - \mu_0 \le 0 \\ & \sum_{i=1}^{m} v_i x_{ik} = 1 \\ & v_i \ge 0; \mu_r \ge 0; \mu_0 \, free \\ & i = 1,2,...,m; r = 1,2,...,q; j = 1,2,...,n. \end{aligned} \tag{3}$$

Model (4) is an output-oriented BCC model:

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} v_i x_{ik} + v_0 \\ s.t. \quad & \sum_{r=1}^{q} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - v_0 \le 0 \\ & \sum_{r=1}^{q} \mu_r y_{rk} = 1 \\ & v_i \ge 0; \mu_r \ge 0; v_0 \, free \\ & i = 1,2,...,m; r = 1,2,...,q; j = 1,2,...,n. \end{aligned} \tag{4}$$

III. A. 2) Interval DEA methods

Interval DEA methods, refers to DEA methods where the DEA efficiency evaluation process contains interval data or where the efficiency measurements are interval numbers. Interval DEA is divided into three main categories: variable substitution methods, interval efficiency methods, and other integrated methods.

The interval efficiency method is often used when the input-output data contains or can be transformed to contain only exact and interval data, and in this paper, the evaluation of piano practicing time uses the interval efficiency method. Firstly, the combination of the indicator data with the minimum input and maximum output of the evaluated DMU and the maximum input and minimum output of other DMUs is solved by deterministic DEA model to get the upper bound of the efficiency of the evaluated DMU. Then the lower bound of efficiency of the evaluated DMU is obtained by substituting the indicator data of the minimum output of the maximum input of the evaluated DMU and the maximum output of the minimum input of the other DMUs into the solution of the DEA model. Finally, the performance performance of the evaluated DMU is represented by the efficiency interval consisting of the upper and lower efficiency bounds. The model of interval efficiency method is shown below.

If the input-output data in model (1) are interval data, i.e., $x_{ij} \in \left[x_{ij}^L, x_{ij}^U\right], y_{rj} \in \left[y_{rj}^L, y_{rj}^U\right]$, then the evaluated DMU's optimal efficiency value can be calculated by model (5):

$$\begin{aligned} \max \quad & h_{j_0}^U = \sum_{r=1}^{s} \overline{\mu}_r y_{rj_0}^U \\ s.t. \quad & \sum_{i=1}^{m} \overline{v}_i x_{ij_0}^L = 1 \\ & \sum_{r=1}^{s} \overline{\mu}_r y_{rj_0}^U - \sum_{i=1}^{m} \overline{v}_i x_{ij_0}^L \le 0 \\ & \sum_{r=1}^{s} \overline{\mu}_r y_{rj}^L - \sum_{i=1}^{m} \overline{v}_i x_{ij}^U \le 0, j = 1,...,n; j \ne j_0 \\ & \overline{\mu}_r, \overline{v}_i \ge \varepsilon, \forall r, i. \\ & i = 1,2,...,m; r = 1,2,...,s; j = 1,2,...,n \end{aligned} \tag{5}$$

The worst efficiency value of the evaluated DMU can be calculated through model (6):

$$\begin{aligned} \max \quad & h_{j_0}^L = \sum_{r=1}^{s} \underline{\mu}_r y_{rj_0}^L \\ s.t. \quad & \sum_{i=1}^{m} \underline{v}_i x_{ij_0}^L = 1 \\ & \sum_{r=1}^{s} \underline{\mu}_r y_{rj_0}^L - \sum_{i=1}^{m} \underline{v}_i x_{ij_0}^L \le 0 \\ & \sum_{r=1}^{s} \underline{\mu}_r y_{rj}^U - \sum_{i=1}^{m} \underline{v}_i x_{ij}^L \le 0, j = 1,...,n; j \ne j_0 \\ & \underline{\mu}_r, \underline{v}_j \ge \varepsilon, \forall r, i. \\ & i = 1,2,...,m; r = 1,2,...,s; j = 1,2,...,n \end{aligned} \tag{6}$$

With the above two models, the interval efficiency of the evaluated DMUs can be obtained, denoted as $\left[h_{j_0}^L, h_{j_0}^U\right]$. Based on the obtained interval efficiency, the DMUs are classified into three categories:

$$E^{++} = \{DMU_j, j = 1,2,...,n \mid h_j^L = 1\} \tag{7}$$

$$E^{+} = \{DMU_j, j = 1,2,...,n \mid h_j^U = 1, h_j^L < 1\} \tag{8}$$

$$E^{-} = \{DMU_j, j = 1,2,...,n \mid h_j^U < 1\} \tag{9}$$

The first class of DMUs in E^{++} has an upper and lower efficiency bound of 1, and the decision unit is said to be fully effective for DEA, the second class of DMUs in E^{+} has an upper efficiency bound of 1 and a lower bound less than 1, and the decision unit is said to be incompletely effective, and the third class of DMUs in E^{-} has an upper and lower efficiency bound of no less than 1, and the decision unit is said to be ineffective for DEA. Due to the interval nature of the resulting efficiency values, it is not possible to directly compare and rank DMUs in the E^{+} and E^{-} classes in terms of relativity, and it is difficult to improve the efficiencies for this type of method.

III. A. 3) Inter-area super-efficiency DEA methodology

In the traditional C^2R model, when the DMUs of multiple camouflage schemes are efficient, there is a problem of not being able to compare these schemes with each other. The reference set of the super-efficient DEA model does not include the evaluated camouflage scheme DMU_0 itself, but compares the evaluated unit with a linear combination of all other evaluated units. The value of the maximum proportion of a DMU that can increase its inputs while maintaining relative effectiveness is referred to as the super-efficiency value, which is obviously likely to be greater than 1. The problem of sufficiently comparing DMUs can be solved by comparing the super-efficiency values obtained by super-efficiencyization in the C^2R model where the result is an effective DMU.

1) Interval super-efficiency DEA modeling

There are n DMUs with m number of input indicators and s number of output indicators. $X_j = (x_{1j}, x_{2j}, \cdots, x_{mj})^T$ is the input dataset, $Y_j = (y_{1j}, y_{2j}, \cdots, y_{ij})^T$ is the output data set, where $j = 1,2,\cdots,n$. Then the super-efficient DEA model (with respect to the DEA (C^2R) model) for the evaluation unit j_0 (subscripted as 0, below) decision unit DMU_0 is:

$$\max h_0 = \sum_{r=1}^{s} \mu_r y_{r0}$$
$$s.t.\begin{cases} \sum_{i=1}^{m} \omega_i x_{i0} = 1 \\ \sum_{i=1}^{m} \omega_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} \ge 0, j = 1,2,\cdots,n(j \ne 0) \\ \omega_i, \mu_r \ge 0, \forall i, r \end{cases} \tag{10}$$

The DEA requires that the inputs and outputs of the DMUs are all accurate data, in which case the super-efficient DEA model is extended interval-wise.

Let $A = \left[a^l, a^U\right] = \left\{x : a^L \le x \le a^U, x \in R\right\}$, and call A an interval number, and a^L, a^U are the lower limit and the upper limit.

When $a^L = a^U$, the interval number A degenerates to a real number.

From the definition of interval number, consider the input and output data as $x_{ij} = \left[x_{ij}^L, x_{ij}^U\right], y_{ij} = \left[y_{ij}^L, y_i^U\right]$, $i = 1,2,\cdots,m, r = 1,2,\cdots,s, j = 1,2,\cdots,n$. The interval super-efficiency DEA model is:

$$\max h_0 = \sum_{r=1}^{s} \mu_r \left[y_{r0}^L, y_{r0}^U\right]$$
$$s.t.\begin{cases} \sum_{i=1}^{m} \omega_i \left[x_{i0}^L, x_{i0}^U\right] = 1 \\ \sum_{i=1}^{m} \omega_i \left[x_{ij}^L, x_{ij}^U\right] - \sum_{r=1}^{s} \mu_r \left[y_{rj}^L, y_{rj}^U\right] \ge 0, j = 1,2,\cdots,n(j \ne 0) \\ \omega_i, \mu_r \ge 0, \forall i, r \end{cases} \tag{11}$$

2) Interval super-efficiency value upper and lower bound solving

Considering the most favorable case for a certain scheme DMU_0, i.e., x_{i0}^L and y_{r0}^U as the input and output values of DMU_0 and x_{ij}^U and $y_{ij}^L (j \ne 0)$ as the input and output values of the other pseudo-schemes of DMUs, model (12) models the upper bound of the interval super-efficiency value of DMU_0. Relatively, considering the most unfavorable case for a certain scheme DMU_0, i.e., x_{i0}^U and y_{r0}^L as the input-output values of DMU_0, and x_{ij}^L and $y_{ij}^U (j \ne 0)$ as the input-output values of the other DMUs, model (13) is a lower bound model for the interval super-efficiency values of DMU_0:

$$\max h_0 = \sum_{r=1}^{s} \mu_r y_{r0}^U$$
$$s.t.\begin{cases} \sum_{i=1}^{m} \omega_i x_{i0}^L = 1 \\ \sum_{i=1}^{m} \omega_i x_{ij}^U - \sum_{r=1}^{t} \mu_r y_{ij}^t \ge 0, j = 1,2,\cdots,n(j \ne 0) \\ \omega_i, \mu_r \ge 0, \forall i, r \end{cases} \tag{12}$$

$$\min h_0 = \sum_{r=1}^{s} \mu_r y_{r0}^L$$
$$s.t.\begin{cases} \sum_{i=1}^{m} \omega_i x_{i0}^U = 1 \\ \sum_{i=1}^{m} \omega_i x_{ij}^L - \sum_{r=1}^{s} \mu_r y_{ij}^U \geq 0, j = 1,2,\cdots,n(j \neq 0) \\ \omega_i, \mu_r \geq 0, \forall i, r \end{cases} \tag{13}$$

Let the upper and lower limit values obtained by model (12) and model (13) be h_0^U and h_0^L, respectively, then the value of the over-efficiency of DMU_0 is the number of intervals $\left[h_0^L, h_0^U\right]$, i.e., it is the interval over-efficiency of the DMU_0. value.

3) DMU classification based on interval super-efficiency value

DMUs are categorized into three classes based on the interval super-efficiency value $\left[h_0^L, h_0^U\right]$ of DMU_0:

(1) $E^+ = \left\{DMU_j \mid h_j^L \geq 1, j = 1,2,\cdots,n\right\}$, and $DMU_0 \in E^+$ is said to be interval DEA efficient if $DMU_0 \in E^+$ is said to be DEA efficient for all inputs and outputs are DEA efficient.

(2) $E = \left\{DMU_i \mid h_i^l < 1 \text{ and } h_i^u \geq 1, j = 1,2,\cdots,n\right\}$, and $DMU_0 \in E$ is said to be partially valid for the interval DEA if DMU_0 is partially valid for the interval DEA, with is DEA efficient for some defined level of input output.

(3) $E^- = \left\{DMU_j \mid h_j^U < 1, j = 1,2,\cdots,n\right\}$, if $DMU_0 \in E^-$, then DMU_0 is said to be the interval DEA is invalid, and it is non-DEA valid for all inputs and outputs. are non-DEA valid. Denote the set of all DMUs as S, and obviously $S = E^+ \cup E \cup E^-$.

4) DMU sufficiently sorted

When a, b are both intervals or one of them is an interval, let $a = \left[a^L, a^U\right], b = \left[b^L, b^U\right]$, then it is said:

$$P(a > b) = \max\left\{1 - \max\left\{\frac{b^U - a^L}{a^U - a^L + b^U - b^L}, 0\right\}, 0\right\} \tag{14}$$

is the likelihood degree (PDF) of $a > b$.

With the above definition, the interval super-efficiency values of all DMUs are compared two by two to establish the likelihood matrix as:

$$P = \begin{bmatrix} 0.5 & p_{12} & \cdots & p_{1n} \\ p_{21} & 0.5 & \cdots & p_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ P_{n1} & P_{n2} & \cdots & 0.5 \end{bmatrix} \tag{15}$$

where p_{ij} denotes the PDF of $p_i > p_j$.

Eq. (15) contains all the information about the likelihood of all the schemes comparing with each other, so the problem of sorting the interval numbers can be transformed into a problem of solving the sorting vector of the likelihood matrix, for the elements $P_{ij}(i, j = 1,2,\cdots,n)$ in the likelihood matrix P satisfy $0 \leq p_{ij} \leq 1$, and $p_{ij} + p_{ji} = 1(i, j = 1,2,\cdots,n)$, so the likelihood matrix P is a fuzzy complementary judgment matrix.

Summing the matrix P by rows yields:

$$r_i = \sum_{j=1}^{n} p_{ij}, (i = 1,2,\cdots,n) \tag{16}$$

Row sum normalization is then performed to find its ordering vector as $W = (w_1, w_2, \cdots, w_n)^T$. Calculate Eq:

$$w_i = \frac{\sum_{j=1}^{n} p_{ij} + \frac{n}{2} - 1}{n(n-1)}, (i = 1,2,\cdots,n) \tag{17}$$

For a fuzzy complementary matrix $P=(p_{ij})_{n\times n}$, for $\forall k$, there is $p_{ik} \geq p_{jk}(p_{ik} \leq p_{jk})$, then $w_i \geq w_j(w_i \leq w_j)$, if and only if all the former equations hold, $w_i = w_j$. Thus, to compare the magnitudes of w_i and w_j, it is sufficient to compare the magnitudes of r_i using the above property.

III. B. Evaluation of time allocation for piano practice

III. B. 1) Interval DEA-based analysis

The data of 21 learners were randomly selected from the piano practice time allocation data collected in the previous section, interval processed, and substituted into the traditional interval DEA model to obtain the evaluation results of piano practice time allocation for each learner as shown in Table 1. In the 21 learners' piano practice time allocation, the 7th, 11th, 16th and 21st decision unit efficiency results are all 1. This decision unit is effective, but it is impossible to distinguish which decision unit is better, and it is impossible to rank the decision units accordingly, based on which the interval super-efficiency model is introduced for evaluation.

Table 1: The evaluation results of the traditional interval DEA model

Number	Evaluation results	Number	Evaluation results	Number	Evaluation results
1	[0.99 1]	8	[0.95 0.98]	15	[0.96 0.98]
2	[0.97 0.99]	9	[0.97 0.99]	16	[1 1]
3	[0.98 0.99]	10	[0.98 0.99]	17	[0.98 0.99]
4	[0.97 1]	11	[1 1]	18	[0.99 1]
5	[0.96 0.98]	12	[0.98 0.99]	19	[0.98 0.99]
6	[0.98 0.99]	13	[0.97 0.99]	20	[0.95 0.98]
7	[1 1]	14	[0.95 0.98]	21	[1 1]

III. B. 2) Analysis based on interval super-efficiency DEA

Substitute the above data into the interval super-efficiency DEA model to obtain the optimal values of the upper limit model and the lower limit model as well as the interval efficiency values of DMU_0, and then classify and sort the interval super-efficiency values. The upper and lower limit values and interval efficiency values of piano practice time allocation are shown in Fig. 3, and the results of interval super-efficiency sorting are shown in Fig. 4. The classification results of piano time allocation are shown in Table 2.From the sorting, the evaluation of the piano practice time allocation of the learner of sample No. 16 is optimal, with an evaluation efficiency of 1.502, which is much better than the other sample interval super-efficiency values, and the classification result is E+, which indicates that his piano practice time allocation is effective. The results of the interval super-efficiency allocation of learners of samples No. 7, No. 11 and No. 21 are also all E+, with evaluation efficiencies amounting to 1.117, 1.084 and 1.235, which ranked No. 2 to 4. Of the 21 learners taken, 11 learners had an E, i.e., interval DEA partially efficient, classification result for piano practice allocated time, which was the largest percentage of learners at 52.38%. There are still 6 learners whose piano practice allocated time classification result is E-, which belongs to the interval DEA invalid category.

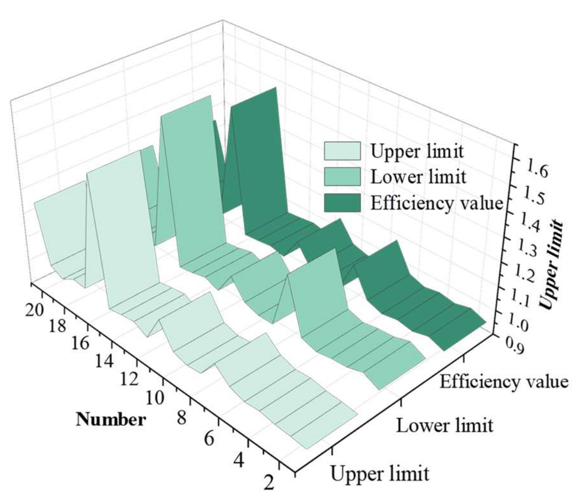


Figure 3: The upper and lower limit and interval efficiency of piano practice time allocation

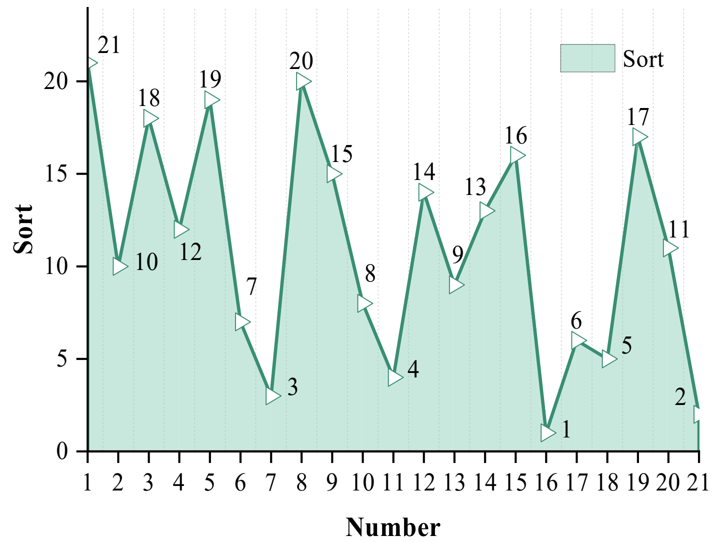


Figure 4: The sorting results of interval hyper efficiency

Table 2: The classification of piano time allocation

Number	Classification	Number	Classification	Number	Classification
1	E	8	E-	15	E-
2	E	9	E-	16	E+
3	E	10	E	17	E
4	E	11	E+	18	E
5	E-	12	E-	19	E-
6	E	13	E	20	E
7	E+	14	E	21	E+

IV. Strategies for improving the effectiveness of piano practice

Learning piano should also master the correct technique practice method, improve the efficiency of piano practice, reflect on the practice gains and losses, and choose reasonable practice methods. Different historical periods, schools, and scholars have different methods of practicing piano technique, so it is necessary to choose specific practice methods from the actual situation to meet the need of practicing piano technique efficiently. Combined with the piano practice survey of learners and the evaluation of piano practice time allocation based on data envelopment analysis, this chapter discusses the strategies to improve the effectiveness of piano practice.

IV. A. Strengthening piano fingering practice

Piano practice should pay particular attention to fingering practice, not only to ensure that fingering is practiced according to a schedule, but also to enhance reflection on fingering practice. First of all, the practitioner should try to figure out the fingering problems by himself, and develop correct fingering habits. Secondly, fingering practice can adopt the method of video analysis and recording to video-record their piano playing, so that they can reflect more profoundly and solve the piano fingering problems in a more targeted way. Once again, piano practice can be used to break up the practice method, focusing on only one hand, you can effectively grasp the pitch, rhythm and fingering of one hand, analyze the problems in playing, so as to focus on breaking through the problems one by one. Piano fingering practice should avoid the vicious circle, to prevent over-practice hand muscle damage.

IV. B. Control the time of practice

Generally speaking, the player's playing ability, physical quality, the technical stage and many other factors will affect the practice time. Reasonable control of practice time, appropriate prolongation or shortening of the practice time, can play twice the result with half the effort. Practitioners should start from the actual situation, according to the needs of the practice program, to develop a scientific and reasonable practice plan, to determine the basic objectives of practice, clear practice of the basic methods of piano skills, and strive to complete the established practice tasks within a limited period of time. In order to eliminate the practitioner's sense of fatigue, but also after a long period of practice to take a full rest, to avoid psychological damage. When encountering more difficult rhythms, the method of decentralized practice should be used, focusing on the time to conquer each bar, so as to improve the overall effect of practice.

IV. C. Focus on technical difficulties

First of all, the goal of practice should be clearly defined, the method of practice should be defined according to the goal of practice, and the desired effect should be analyzed to see if it is achieved in the process of practice. Secondly, the piano practitioner should also be good at deep understanding, actively analyze the gains and losses of practice, and deeply experience the emotion of the work when practicing, so as to reduce the waste of time and reduce the intensity of practice. Thirdly, the practitioner should also try to think about the characteristics of the piece after each practice, so as to grasp the state of the piece, and to achieve the goal of improving the piano skills in a comprehensive way.

IV. D. Actively drawing on the experience of others

Piano technique practice should also draw extensively on the experience of others, seriously summarize the gains and losses of others' piano practice, and learn from the masters' piano practice methods, so as to improve the overall level of piano practice by constantly learning from their strengths and making up for their weaknesses. In order to improve the efficiency of piano practice, some pianists adopt the methods of practicing on the piano by reading the score, practicing without reading the score, reading the score without playing the piano, etc., and strive to combine the understanding of the work with the piano practice, so as to achieve a better practice effect.

V. Conclusion

The scientific evaluation of piano practice time allocation and the development of optimization strategies are of great practical significance. Through the analysis of interval super-efficiency DEA model, it was found that only 4 learners (19.05%) out of 21 learners in the sample had interval DEA effective evaluation results of piano practice time allocation, and the first ranked learner had a super-efficiency value as high as 1.502, which was significantly better than the other learners. Most of the learners (52.38%) had a partially effective interval DEA for piano practice time allocation and needed to make optimized adjustments in their practice methods. The questionnaire survey showed that 62.66% of the learners used the centralized practice method, only 34.26% of the learners could insist on practicing the piano every day, and the percentage of those who did hand muscle practice and technique practice before practicing the piano was only 25.98%. To address these problems, piano learners should strengthen fingering practice and reflection, adopt video analysis and recording method and break-up practice method to improve fingering skills; scientifically control the practice time according to their personal situation, and adopt a combination of centralized and decentralized practice; clearly define the practice goals and focus on the technical difficulties; actively learn from the experience of masters, and try to practice on the piano by reading the score, practicing without reading the score and other diversified methods. The comprehensive application of these enhancement strategies will significantly optimize the practice effect and improve the practice efficiency of piano learners.

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