International Journal for Housing Science and Its Applications
Publish August 6, 2025. Volume 46, Issue 3 Pages 7124-7138

https://doi.org/10.70517/ijhsa463612

Research on Topology Optimization Modeling of Complex Corporate Financial Networks and Improvement of Corporate Comprehensive Efficiency

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Abstract The trend of conglomerate and globalization in enterprise development not only complicates the enterprise financial network, but also makes capital flow, equity relationship and resource allocation efficiency gradually become the key factors affecting the comprehensive performance of enterprises. In this paper, from the financial and non-financial perspectives, we initially design 36 corporate financial risk early warning indicators under 9 different dimensions. Then it proposes a method to mine higher-order topological features in relational networks to improve the prediction accuracy of corporate financial risk dilemmas. The method constructs higher-order topological structure features by constructing a heterogeneous graph representation learning model, and identifies the higher-order topological structures that occur frequently and have an important impact on corporate financial risk. Subsequently, the weights and thresholds of the BP neural network are optimized by using the Tennessee whisker search method, so as to establish the financial risk early warning system based on BAS-BP neural network and complete the construction of the enterprise financial risk early warning model. At the same time, the 36 selected three-level indicators were tested for normality and non-parametric test, and finally the enterprise financial risk early warning indicators with 8 common factors as the main components and 27 three-level indicators were established. The proposed model is used to predict the financial fraud risk of enterprise P during a total of five years from 2017 to 2021, and its prediction results have a coefficient of determination greater than 0.900, and the root-mean-square error is less than 0.200, which is much better than similar modeling methods.

Index Terms heterogeneous graph representation learning, financial risk early warning, BAS-BP neural network, aspersion whisker search

I. Introduction

The arrival of economic globalization, every country will inevitably be affected by it, especially on the banking industry is very big, most of the banks will take the way to reduce the loan to protect their own interests [1], [2]. In the face of such a situation, those who carry out technological transformation, industrial upgrading needs a lot of money to protect the enterprise, at this time the financing, investment, cash flow encountered very big difficulties [3]-[5]. And the acceleration of the flow of funds, so that enterprises have higher requirements for the utilization efficiency of fast-moving financial information. The financial operation of enterprises presents a complex network model, which is no longer limited to a single purchase and sale transaction in the past, but also includes cross-border capital flows, inter-enterprise mergers and acquisitions activities, etc., as well as the proliferation of financial nodes, the complexity of capital flow channels, and the periodic re-adjustment of the financial structure [6]-[9].

In the era of high technological development, information technology has also become a financial management tool that companies rely on. The original decentralized and high-cost financial management model can no longer meet the needs of enterprises, because it contains a large number of duplication of efforts, resource-consuming and inefficient, so the transformation of the financial management model for the business complex enterprise needs [10]-[12]. The advantages and disadvantages of an enterprise's financial strategic management not only determines the enterprise's ability to deal with economic globalization, but also relates to the survival of the enterprise [13]. Therefore, the financial strategic management of an enterprise becomes more and more important. The limitations of the traditional financial management model in dynamic demand changes, multi-source data heterogeneity, risk management, etc. are highlighted, which reduces the efficiency of financial management and further reduces the comprehensive efficiency of enterprises [14]-[16].

Topology optimization is a method used to solve topology optimization problems in graph theory. Its application areas include telecommunication networks, the Internet, data center networks, wireless sensor networks, industrial control networks, intelligent transportation networks, smart grids, etc., where it can improve the communication quality and data transmission speed of the network, reduce the energy consumption and cost of the network, enhance the security and reliability of the network, and satisfy the needs of different applications [17]-[20]. Through topology optimization modeling of financial networks, we seek dynamic planning paths for financial networks, as well as cash flow optimization and capital turnover, by adjusting and improving the nodes and links in the network in order to improve the performance, reliability, efficiency, and security of financial networks [21], [22].

This paper firstly combines the existing research and the actual situation of enterprises to sort out a total of 36 enterprise financial risk early warning level 3 indicators from the financial and non-financial perspectives. Secondly, it elaborates the construction method based on heterogeneous graph representation learning topological structure features as the warning algorithm of enterprise financial risk warning model. At the same time, we analyze the optimization process of BP neural network weights and thresholds based on the Tennessee whisker search method, and establish the BAS-BP model. Integrate the learning topological structure features based on heterogeneous graph representation to build the enterprise financial risk early warning model comprehensively. The structure and content of the three-level indicators of corporate financial risk early warning are optimized in the form of normality test and non-parametric test, and factor analysis is carried out on the final 27 three-level indicators of corporate financial risk early warning. Finally, the designed model is trained and optimized, and the performance of the model is evaluated by comparing the algorithms of similar models. Based on the content and results of the analysis, suggestions are made to improve the comprehensive efficiency of enterprises.

II. Selection of Early Warning Indicators for Enterprise Financial Risks

Comprehensive existing research results, financial indicators can more directly reflect the enterprise's financial risk status, and non-financial indicators are also an important guarantee of its healthy development status, so this paper from the financial indicators and non-financial indicators on the selection of financial early warning indicators.

(1) Financial indicators

1) Solvency

In evaluating the financial status and operating results of an enterprise, the first thing to consider is the solvency of the enterprise, which is also an important factor reflecting whether the enterprise can continue to operate. In this paper, five indicators are selected in terms of solvency, including current ratio, quick ratio, cash current liability ratio, gearing ratio and equity ratio.

2) Profitability

Profitability is an ability that enterprise managers, creditors, investors and all employees are most concerned about. It is the most direct reflection of the enterprise after the production and operation, whether the enterprise operation is effective depends on how the profitability of the enterprise. The profitability indicators selected in this paper are operating profit margin, return on assets, return on net assets, gross profit margin, sales period expense rate, cost and expense margin and asset impairment operating income seven indicators.

3) Development ability

Development ability considers the enterprise's planning and response ability for the future. Whether it can attract investors to support the enterprise, the enterprise's own development ability is crucial. The development ability indicators selected in this paper include five indicators: net profit growth rate, total asset growth rate, operating profit growth rate, sustainable growth rate and capital preservation and appreciation rate.

4) Operating capacity and cash flow

The operating capacity of an enterprise refers to the enterprise's ability to raise a certain amount of funds, which will be used in all the daily production and operation activities of the enterprise, so that the enterprise can operate safely and smoothly, and the internal funds can turn over quickly, which will ultimately produce a certain amount of benefits, and enable the enterprise to develop healthily and sustainably. The evaluation indexes of operating ability and cash flow selected in this paper are five indexes: accounts receivable turnover, inventory turnover, current asset turnover, net cash flow from operating income and cash meets investment ratio.

(2) Selection of non-financial indicators

Previous research by scholars has proved that the appropriate inclusion of non-financial indicators in the early warning model can effectively improve the accuracy of the model. When establishing the early warning model, it is necessary to comprehensively consider the factors affecting the occurrence of financial risk of enterprises, and to select appropriate non-financial indicators. In this paper, the selection of non-financial indicators is carried out from the aspects of corporate governance structure, shareholding structure, significant events, audit, goodwill and equity pledge.

1) Corporate governance structure

Corporate governance structure and financial risk are related to some extent. The rationalization of corporate governance structure helps to improve the management and development of enterprises. Therefore, corporate governance indicators are considered to be introduced into the financial crisis early warning indicator system. This paper chooses two indicators, whether the chairman and general manager are concurrently appointed and whether the corporate executives are changed.

Table 1: Financial risk early warning indicators

<table>
<tr><th>Primary index</th><th>Secondary index</th><th>Three-level index</th></tr>
<tr><td rowspan="22">(A)Financial indicators</td><td rowspan="5">(A1)Solvency</td><td>(A11)liquidity ratio</td></tr>
<tr><td>(A12)quick ratio</td></tr>
<tr><td>(A13)cash coverage ratio</td></tr>
<tr><td>(A14)asset-liability ratio</td></tr>
<tr><td>(A15)equity ratio</td></tr>
<tr><td rowspan="7">(A2)Profitability</td><td>(A21)return on assets</td></tr>
<tr><td>(A22)return on equity</td></tr>
<tr><td>(A23)gross profit margin</td></tr>
<tr><td>(A24)operating margin</td></tr>
<tr><td>(A25)Expense ratio during the sales period</td></tr>
<tr><td>(A26)ratio of profits to cost</td></tr>
<tr><td>(A27)The ratio of asset impairment to operating income</td></tr>
<tr><td rowspan="5">(A3)Develop capabilities</td><td>(A31)Capital preservation and appreciation rate</td></tr>
<tr><td>(A32)Growth rate of total assets</td></tr>
<tr><td>(A33)net profit growth rate</td></tr>
<tr><td>(A34)Growth rate of operating profit</td></tr>
<tr><td>(A35)sustainable growth rate</td></tr>
<tr><td rowspan="5">(A4)Operating capacity and cash flow</td><td>(A41)turnover of account receivable</td></tr>
<tr><td>(A42)rate of stock turnover</td></tr>
<tr><td>(A43)velocity of liquid assets</td></tr>
<tr><td>(A44)Net cash content of operating income</td></tr>
<tr><td>(A45)Cash meets the investment ratio</td></tr>
<tr><td rowspan="14">(B)Non-financial indicators</td><td rowspan="3">(B1)Equity structure</td><td>(B11)share proportion of the largest shareholder</td></tr>
<tr><td>(B12)H exponent</td></tr>
<tr><td>(B13)Z exponent</td></tr>
<tr><td rowspan="2">(B2)Litigation and arbitration</td><td>(B21)appeal to arbitration</td></tr>
<tr><td>(B22)The number of lawsuits and arbitrations</td></tr>
<tr><td rowspan="2">(B3)Corporate governance structure</td><td>(B31)The situation where the chairman and the general manager hold concurrent positions</td></tr>
<tr><td>(B32)top management turnove</td></tr>
<tr><td rowspan="2">(B4)Audit</td><td>(B41)type of audit opinion</td></tr>
<tr><td>(B42)auditor change</td></tr>
<tr><td rowspan="3">(B5)Goodwill</td><td>(B51)Whether the goodwill is impaired</td></tr>
<tr><td>(B52)Added value of goodwill</td></tr>
<tr><td>(B53)goodwill impairments</td></tr>
<tr><td rowspan="2">(B6)Equity pledge</td><td>(B61)Whether equity pledge has been newly added</td></tr>
<tr><td>(B62)Equity pledge ratio</td></tr>
</table>

2) Equity structure

The equity structure of a company has a great impact on the operation and management of the company. Equity is mainly divided into two kinds of equity concentration and equity dispersion, and it is found that the increase in the proportion of shares held by the major shareholders will lead to unfairness and increase the risk of the enterprise's share price collapse. Excessive shareholding by the first major shareholder will result in excessive shareholder rights and interference in the development of the enterprise. The introduction of equity checks and balances is conducive to the stable development of the enterprise.

3) Audit Opinion

Audit is a means of predicting financial risk. Listed companies have an accounting firm audit their finances every year and issue an audit report. If a standard unqualified opinion is issued, it proves that the probability of the enterprise's financial risk is small. 4) Goodwill impairment

Goodwill impairment further affects the growth of a business by affecting its profits and share price volatility. Many enterprises due to high performance commitments in the process of mergers or acquisitions such as high premium mergers and acquisitions, resulting in the enterprise goodwill scale is inflated, and in the later stage of operation due to the emergence of problems and the emergence of large amounts of goodwill impairment. The risk of impairment should not be underestimated.

5) Equity Pledge

Equity pledge is the stock as the subject matter, so as to exchange the behavior of funds. After the shareholders make equity pledge, in order to avoid a continuous decline in share price or other adverse effects, they will purposefully intervene in the company's business decision-making program and major matters, which may further lead to financial risk.

To summarize the above, the preliminary selection of financial risk early warning indicators is shown in Table 1.

III. Early warning model for enterprise financial risk

III. A. Higher-order topology construction based on heterogeneous graph representation learning

The goal of the unsupervised heterogeneous graph representation learning model is to construct higher-order topology features without using corporate financial distress labeling information, i.e., by graph comparison learning to train the encoder $f:\mathrm{V}\to\square^{d}$, which encodes the higher-order topology in which each entity $e\in E$ in G is located as a d-dimensional vector e_t. The unsupervised heterogeneous graph representation learning model consists of three modules in which network data augmentation generates two network views by a perturbation method. The heterogeneous attention mechanism-based encoder constructs higher-order topological structure features from the heterogeneous network by improving the existing attention mechanism to recognize different types of entities and associative relationships. The unsupervised loss function guides the model to update the parameters through a similarity strategy, i.e., maximizing the similarity of the same higher-order topological structure features in both network views.

(1) Network Data Enhancement

Network data augmentation aims to generate two network views that match the distribution of the relationship network data of NSS enterprises. Common network data enhancement methods include entity perturbation method, association relationship perturbation method and entity attribute masking method. Since entities and association relationships are the components of higher-order topologies, the entity perturbation method and association relationship perturbation method are selected for network data enhancement in this chapter. Specifically, perturbation is applied to entities and association relations in the relationship network of NSS enterprises according to Bernoulli distribution, and some entities and association relations are randomly deleted to generate two network views.

(2) Encoder based on heterogeneous attention mechanism

Due to the heterogeneous characteristics of the heterogeneous network of NSSB enterprises, changes in any of the entities and affiliations will cause changes in the higher-order topology, and the impact of different higher-order topologies on the financial distress of enterprises also changes. Therefore, modeling higher-order topology needs to consider the interaction between entity types and associated relationship types. Most of the existing attention mechanisms are oriented to different types of entities or different types of affiliations, and lack the ability to capture different types of entities and different types of affiliations simultaneously, making it difficult to accurately portray the higher-order topology in the heterogeneous networks of NSSEs. For this reason, this paper proposes a heterogeneous attention mechanism, constructs a graph neural network layer, and then builds an encoder based on the heterogeneous attention mechanism shown in Fig. 1. Compared with the existing attention mechanisms, the heterogeneous attention mechanism designs a heterogeneous attention function and a heterogeneous information transfer function for the interactions between the types of entities and types of associative relationships as shown in Eq. (3) and Eq. (5), respectively. Next, the encoder based on the heterogeneous attention mechanism is described in detail.

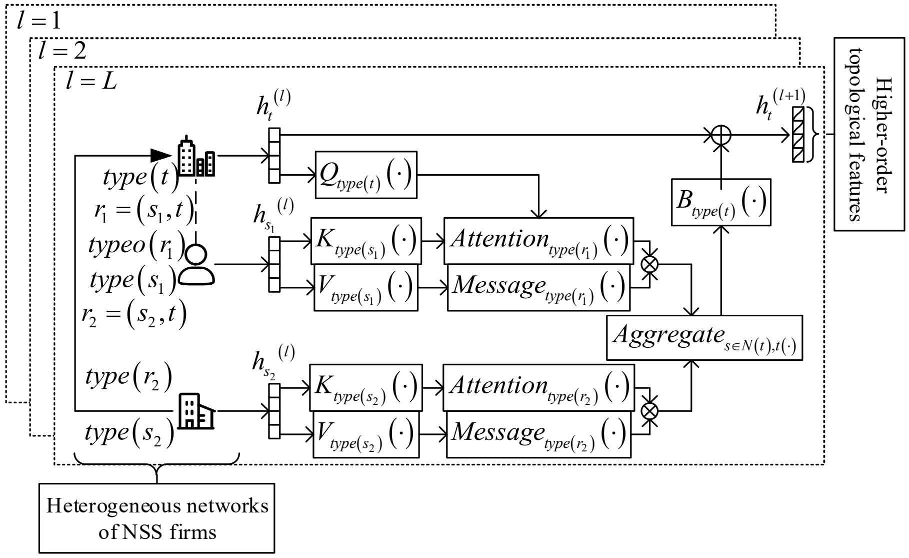


Figure 1: Heterogeneous Attention Based Encoder

The encoder based on the heterogeneous attention mechanism consists of multiple layers of graph neural networks. Each layer of graph neural network mainly consists of three functions: the heterogeneous attention function $\left(Attention_{type(r)}(\cdot)\right)$, the heterogeneous message passing function $\left(Message_{type(r)}(\cdot)\right)$, and the information aggregation function $\left(Aggregate_{s\in N(t),t}(\cdot)\right)$, as shown in shown in Equation (3), Equation (5) and Equation (6).

Given that different types of entities are in different types of feature spaces, the lth layer graph neural network first maps the different types of entities into the same vector space through the mapping function of entity type perception, as shown in Equation (1) and Equation (2). The mapping function consists of a single layer perceptron.

$$Q_t^i = Q_{type(t)}^i(h_t^{(l)}) \tag{1}$$

$$K_s^i = K_{type(s)}^i(h_s^{(l)}) \tag{2}$$

where t represents the target entity and s represents the one-time neighbors of t. $h_t^{(l)}$ represents the higher-order topological features of t in the lth layer of the graph neural network, and $h_t^{(0)}$ represents the random initialization information of t. $Q_{type(t)}^i$ and $K_{type(s)}^i$ represent mapping functions for entity type perception, $type(t), type(s) \in TE$, respectively. The i represents the number of heads in the multi-head attention mechanism.

In order to capture the interaction between entity types and association relation types, the lth layer graph neural network models the attention between two entities under different types of association relation conditions as a way to capture different types of entities and different types of association relations simultaneously, and to quantify the importance of a one-time neighbor to the target entity as shown in Eq. (3).

$$Attention(s,t) = Soft\max_{s\in N(t)}\left(\Big\|_i \frac{K_s^i W_{type(r)}^{ATT} Q_t^i}{\sqrt{d}}\right) \tag{3}$$

where d is the dimension of $h_t^{(l)}$ used for normalization. $type(r) \in TR$, $W_{type(r)}^{ATT}$ represents the mapping matrix for the type-awareness of the association relation, which consists of a matrix with vector dimension $\left(d/\max(i)\right)\times\left(d/\max(i)\right)$. $N(t)$ represents the set of one-time neighbors of t. $\|_i$ represents the splicing notation of the multi-head attention mechanism. $Soft\max$ is the activation function.

Similar to the heterogeneous attention function, the l th layer graph neural network also considers both the entity type and the association relationship type when encoding one degree neighbor information, as shown in Equation (4) and Equation (5).

$$V_s^i = V_{type(s)}^i (h_s^{(l)}) \tag{4}$$

$$Message(s,t) = \|_i V_s^i W_{type(r)}^{MSG} \tag{5}$$

where $V_{type(s)}^i$ represents the entity type-aware mapping function. $W_{type(r)}^{MSG}$ represents the mapping matrix for association relation type perception, which is given by.

In order to aggregate the once-neighborhood information of t, the l th layer graph neural network updates the graph embedding features of t using the information aggregation function as shown in Equation (6).

$$\tilde{h}_t^{(l+1)} = \sum_{s \in N(t)} Attention(s,t) \square\ Message(s,t) \tag{6}$$

Finally, the l-layer graph neural network fuses t's own information using residual connectivity, which is common in deep learning, to obtain the result of the $l+1$-layer graph neural network's encoding of the higher-order topology in which t is embedded, i.e., the higher-order topological features, as shown in Equation (7).

$$h_t^{(l+1)} = B_{type(t)}\left(\sigma\left(\tilde{h}_t^{(l+1)}\right)\right) + h_t^{(l)} \tag{7}$$

where $B_{type(t)}$ represents the mapping function oriented to the target entity t for transforming $\sigma\left(\tilde{h}_t^{(l+1)}\right)$ into the same vector space as $h_t^{(l)}$.

As l increases, the encoder based on the heterogeneous attention mechanism is able to adaptively aggregate information about association relations over long distances, thus encoding the higher-order topology in which the entities are located as vectors, i.e., higher-order topological features.

(3) Unsupervised loss function

Compared to the cross-entropy loss function in supervised graph representation learning models, the contrast learning loss function can utilize network information independent of labeling information to guide the model to update parameters. A representative contrastive learning loss function is the mutual information loss function, i.e., maximizing the mutual information between semantically similar entities (a.k.a., positive example pairs) and minimizing the mutual information between semantically unrelated entities (a.k.a., negative example pairs). However, constructing negative example pairs requires additional design of the model, which leads to an increase in model complexity, and constructing negative example pairs easily destroys the relationship between entities, which affects the model training effect. Therefore, in this study, an unsupervised loss function based on typical correlation analysis is used to construct a heterogeneous graph representation learning model, as shown in Equation (8). Among them, the first loss function aims to maximize the similarity of the same higher-order topological structure features in the two network views, and the goal of the last two loss functions is to minimize the correlation between the dimensions in the higher-order topological structure features, which circumvents the problem of constructing negative example pairs.

$$l = l\left(\left\|\tilde{Z}_A - \tilde{Z}_B\right\|_F^2\right) + \lambda\left(l\left(\left\|\tilde{Z}_A^T \tilde{Z}_A - I\right\|_F^2\right) + l\left(\left\|\tilde{Z}_B^T \tilde{Z}_B - I\right\|_F^2\right)\right) \tag{8}$$

where $\tilde{Z}_A = \frac{Z_A - \mu(Z_A)}{\sigma(Z_A) * \sqrt{N}}$, $Z_A = f_\theta\left(\tilde{G}_A\right)$. $\tilde{Z}_B = \frac{Z_B - \mu(Z_B)}{\sigma(Z_B) * \sqrt{N}}$, $Z_B = f_\theta\left(\tilde{G}_B\right)$. $Z_A = f_\theta\left(\tilde{G}_A\right)$, $Z_B = f_\theta\left(\tilde{G}_B\right)$, $\tilde{G}_A$ and $\tilde{G}_B$ are the two network views obtained by network data augmentation, and $f_\theta(\cdot)$ is an encoder based on the heterogeneous attention mechanism. λ denotes the non-negative hyperparameter that balances the two loss terms.

In Equation (8), the size of λ has a large effect on the unsupervised loss function. When λ is large, the unsupervised heterogeneous map indicates that the learning model emphasizes on reducing the correlation between the individual dimensions of the higher-order topological features. Conversely, unsupervised heterogeneous maps indicate that the learning model focuses on the similarity of the same higher-order topological structure features in both network views.

III. B. Tenebrous whisker search

BAS is a bionic intelligent optimization algorithm inspired by the foraging behavior of the aspen, which has the advantages of simple algorithmic principle, few parameters and fast convergence speed compared with other intelligent optimization algorithms. Figure 2 shows the foraging process of aspen.

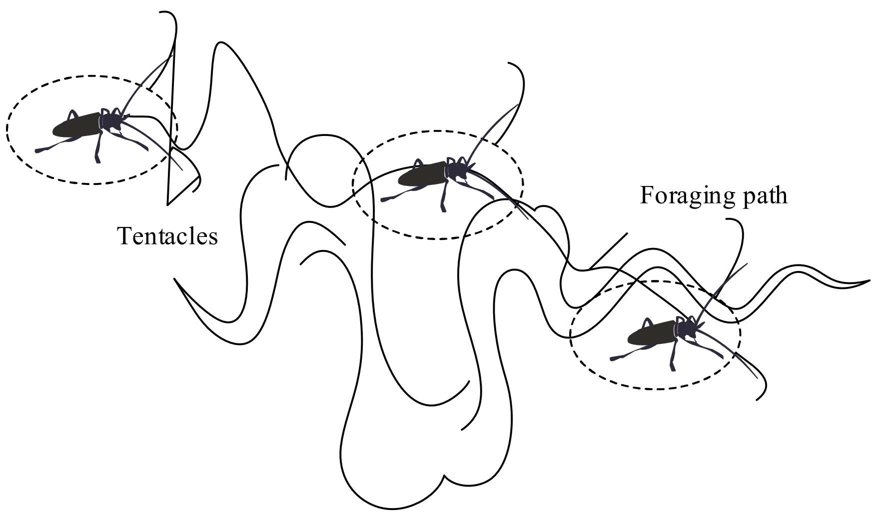


Figure 2: The foraging process of longhorn beetles

Tenebrous foraging relies on two antennae, which sense food odors to determine the path of foraging. During the foraging process, if the left antennae perceive a greater concentration of food odor, then the antennae move to the left, and if the right antennae perceive a greater concentration of food odor, then the antennae move to the right. If the concentration of food odor perceived by the right tentacle is greater, then the tenebrion moves to the right.The BAS makes a reasonable assumption about the tenebrion foraging process, i.e., the tenebrion's two tentacles are on either side of the center of mass, and its head faces randomly for each step it takes while the ratio of the length of the foraging step to the distance of the tentacles is a constant value.The steps of the BAS are:

(1) Initialize the position of the aspens, including the position and distance of the left and right whiskers of the aspens, the position of the center of mass, and the step size, which are x_l, x_r, d_0, x, S_{step} respectively. The unit random vector d_{dir} is generated in MATLAB to represent the orientation of the head of the tenebrae, then there is equation (9):

$$\begin{cases} x_l = x + d_0 \cdot \dfrac{d_{dir}}{2} \\ x_r = x - d_0 \cdot \dfrac{d_{dir}}{2} \end{cases} \tag{9}$$

(2) Construct an iterative format to simulate the aspen foraging process, which is mathematically modeled as equation (10):

$$x^{t+1} = \begin{cases} x^t + S_{step} \cdot d_{dir}(x_l - x_r) & F_l < F_r \\ x^t - S_{step} \cdot d_{dir}(x_l - x_r) & F_l > F_r \end{cases} \tag{10}$$

where, $F_l = f\left(x_l\right)$, $F_r = f\left(x_r\right)$, where $f(\cdot)$ is the function to be optimized.

(3) Determine whether the maximum number of iterations is reached or the set optimization accuracy is satisfied, if not continue to iterate and output the optimized result if satisfied.

III. C. BAS-BP model

Enterprise financial risk is affected by many factors, and BP neural network can be used to provide intelligent early warning of enterprise financial risk. Traditional BP neural networks are more affected by the initial weights and thresholds, which directly affect the performance of prediction.BAS-BP model is a model that combines BAS and BP to optimize the weights and topology of BP neural networks. The model randomly generates a set of initial BP neural network topologies, including the number of network layers, the number of neurons per layer, and the connection weights. To reduce the training error and improve the network training efficiency, the sample data are normalized. The BAS is initialized to determine the fitness function, and the spatial position of the tentacle whiskers is updated by calculating the fitness function value of the tentacle at the initial position according to the fitness function. Compare the fitness function values of the tentacles on both sides of the aspens to determine the direction of the aspens' movement, that is, adjust the weights and thresholds of the BP neural network, and calculate the fitness function value under the current position. Judge whether the fitness function value or the number of iterations meets the set iteration termination conditions, and output when the iteration termination conditions are met to get the optimized initial weights and thresholds, and then get the optimized BP neural network. The BAS is used to optimize the initial weights and thresholds of the BP neural network to achieve the purpose of intelligent early warning of enterprise financial risk. Figure 3 shows the BAS-BP model flow.

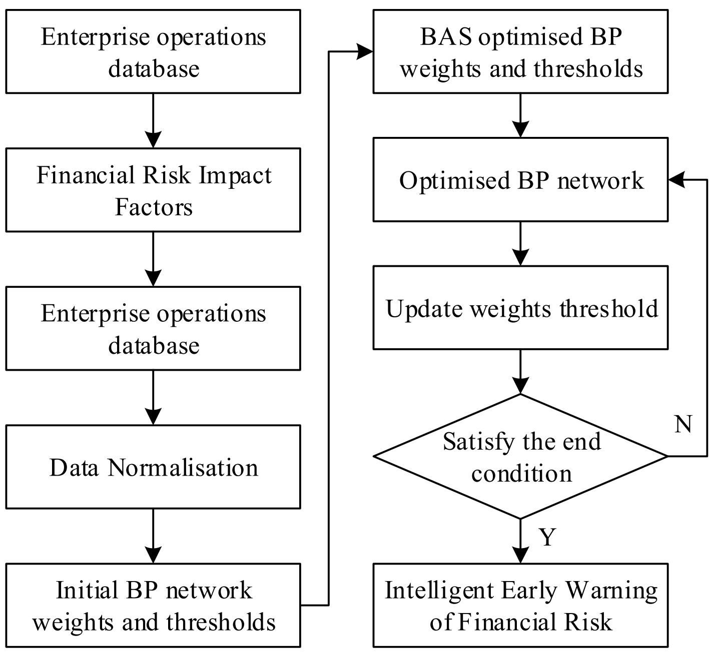


Figure 3: The process of BAS-BP neural network model

IV. Screening and Determination of Early Warning Indicators for Enterprise Financial Risks

IV. A. Normality test

In order to test whether the selected financial warning indicators have significant differences between healthy enterprises and financial warning enterprises, this paper needs to carry out a significance test on the initial indicators, excluding the indicators that do not pass the significance test. Before conducting the indicator significance test need to test the normal distribution of the indicator, if the indicator meets the normal distribution then use the T-test, if it does not meet the normal distribution, then use the non-parametric test for significance analysis.

This paper verifies the normal distribution of the initial indicators through the single-sample K-S test of SPSS software. Firstly, certain enterprises with more missing data are excluded, leaving 167 sample enterprises with a total of 684 data from 2017-2021. Table 2 shows that the significance level of the 36 initial three-level indicators is less than 0.05, which indicates that all indicators do not conform to normal distribution. Therefore, it is necessary to analyze the significance of these initial indicators with nonparametric tests.

Table 2: Kolmogorov-Smirnov inspection

Index	Test statistics	Sig (Double Tail)
A11	0.164	0.000
A12	0.181	0.000
A13	0.196	0.000
A14	0.392	0.000
A15	0.176	0.000
A21	0.007	0.000
A22	0.075	0.000
A23	0.182	0.000
A24	0.064	0.000
A25	0.188	0.000
A26	0.41	0.000
A27	0.16	0.000
A31	0.241	0.000
A32	0.383	0.000
A33	0.318	0.000
A34	0.186	0.000
A35	0.173	0.000
A41	0.444	0.000
A42	0.434	0.000
A43	0.178	0.000
A44	0.233	0.000
A45	0.388	0.000
B11	0.346	0.000
B12	0.442	0.000
B13	0.003	0.000
B21	0.034	0.000
B22	0.246	0.000
B31	0.115	0.000
B32	0.003	0.000
B41	0.285	0.000
B42	0.1	0.000
B51	0.419	0.000
B52	0.127	0.000
B53	0.073	0.000
B61	0.25	0.000
B62	0.283	0.000

IV. B. Non-parametric tests

Non-parametric test for the above early warning indicators that do not meet the normal distribution, if the significance level is less than 0.05 means that it passes the significance test, otherwise it does not pass the test. The results of nonparametric test are shown in Table 3, among which (A15) equity ratio, (A26) cost and expense margin, (A27) asset impairment operating income ratio, (A35) sustainable growth rate, (A45) cash to meet the investment ratio, (B31) chairman of the board of directors and general manager of the concurrently held positions, (B41) the type of audit opinion, (B42) the change of the auditor, (B51) goodwill is or is not A total of nine indicators are greater than 0.05 did not pass the significance test, so excluded. After the above significance test analysis, 27 financial early warning indicators were obtained.

Table 3: Non-parametric test

Index	Mann - Whitney U	Wilcoxon W	Z	Sig (Double Tail)
A11	9943.65	132328.197	-4	0.000
A12	11552.355	58193.211	-6.2	0.000
A13	11957.083	47245.978	-1.14	0.000
A14	5400.227	158645.593	-9.6	0.000
A15	5091.892	181140.806	-3.85	0.089
A21	6699	85809.553	-8.61	0.000
A22	10317.466	6760.168	-13.34	0.000
A23	7633.735	187292.356	-1.24	0.000
A24	11744.936	75698.56	-9.15	0.000
A25	9929.381	100701.994	-3.01	0.000
A26	5627.4	115992.365	-3.4	0.011
A27	6818.758	75531.675	-11.45	0.036
A31	10184.958	190807.166	-5.67	0.000
A32	10437.141	124337.494	-7.29	0.000
A33	8156.574	191990.342	-10.33	0.000
A34	8160.117	175731.692	-7.57	0.000
A35	10189.094	129433.883	-14.53	0.033
A41	5545.121	131344.428	-2.54	0.000
A42	8343.021	140111.724	-10.45	0.000
A43	6517.225	55079.157	-8.61	0.000
A44	6172.096	95389.182	-12.06	0.000
A45	8662.472	108309.244	-10.6	0.085
B11	10471.561	170511.8	-9.96	0.000
B12	8393.189	195555.558	-3.59	0.000
B13	11192.105	37748.238	-5.19	0.000
B21	9441.648	117098.381	-4.03	0.000
B22	11265.261	170372.266	-0.25	0.000
B31	6058.156	41845.509	-1.58	0.019
B32	9656.649	162252.788	-7.83	0.000
B41	9806.841	87823.996	-8.75	0.065
B42	7032.287	121879.764	-0.39	0.028
B51	11468.427	88183.339	-8.27	0.096
B52	4467.823	109391.041	-11.62	0.000
B53	5573.644	176962.503	-11.26	0.000
B61	6274.105	189132.628	-0.8	0.000
B62	8199.216	83571.248	-9.91	0.000

IV. C. Factor analysis

Using SPSS software to factor analyze the 27 indicators to get the total variance interpretation results are shown in Table 4, where “a” is the initial eigenvalue variance percentage (%), “b” is the extracted loadings squared and variance percentage (%), “c” is the rotated load squared and variance percentage (%) It can be found that the cumulative explanation of the first 8 factors is 81.906% and the eigenvalue of the common factor is greater than 1. Therefore, 8 common factors are extracted from the 27 early warning indicators.

According to the load of variables in the factors, the following 8 factors are extracted: (F1) solvency, (F2) profitability, (F3) development ability, (F4) operating capacity and cash flow, (F5) equity structure, (F6) litigation and arbitration, (F7) goodwill, (F8) equity pledge, among which the senior management of the third-level index (B32) is changed to the equity structure of the second-level index (B1), that is, the equity structure of the public factor (F5). The matrix of component score coefficients is shown in Table 5.

Table 4: Explanation of total variance

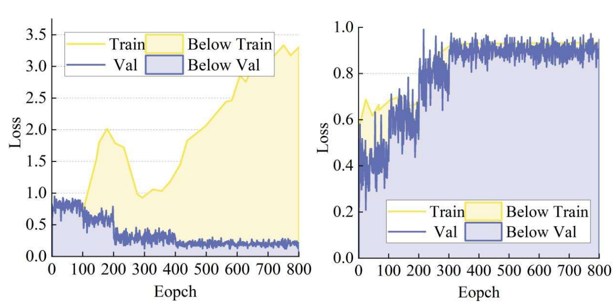
Index	Total	a	Accumulation (%)	Total	b	Accumulation (%)	Total	c	Accumulation (%)
A11	2.707	38.968	38.968	2.707	38.968	38.968	2.821	25.861	25.861
A12	2.6	24.488	63.456	2.6	24.488	63.456	2.345	14.945	40.806
A13	2.433	7.751	71.207	2.433	7.751	71.207	2.159	13.364	54.17
A14	3.42	4.257	75.464	3.42	4.257	75.464	2.006	10.01	64.18
A21	2.122	2.217	77.681	2.122	2.217	77.681	1.26	5.058	69.238
A22	1.86	1.465	79.146	1.86	1.465	79.146	1.117	2.315	71.553
A23	1.787	1.382	80.528	1.787	1.382	80.528	1.084	1.781	73.334
A24	0.992	1.378	81.906	1.992	1.378	81.906	1.001	1.541	74.875
A25	0.942	1.357	83.263						
A31	0.785	1.252	84.515						
A32	0.779	1.227	85.742						
A33	0.718	1.211	86.953						
A34	0.711	1.186	88.139						
A41	0.599	1.141	89.28						
A42	0.564	1.13	90.41						
A43	0.523	1.126	91.536						
A44	0.509	1.125	92.661						
B11	0.284	1.049	93.71						
B12	0.24	1.044	94.754						
B13	0.231	1.037	95.791						
B21	0.201	0.947	96.738						
B22	0.199	0.92	97.658						
B32	0.187	0.82	98.478						
B52	0.048	0.503	98.981						
B53	0.007	0.359	99.34						
B61	0.992	0.343	99.683						
B62	0.942	0.317	100						

V. Integrated Efficiency Improvement Strategies Based on Financial Risk Early Warning

V. A. Model Training and Optimization

V. A. 1) Sample Training and Simulation Testing

In this paper, through a large number of tests, we gradually determine the parameter combinations when the indicators of the enterprise financial risk early warning model are optimal, and finally determine the number of hidden layers as two layers, the number of nodes in each layer is 130 and 65, the learning rate is 0.002, and the number of training times is set at 800 times. Figure 4 shows the training process of the model precision rate, recall rate, F1 value and support metrics, it can be seen that the model fits better, the loss on the training set steadily decreases to the vicinity of 0, and the precision rate, recall rate, and AUC (area under the curve) are continuously improved to 1. The loss on the validation set is finally 3.41, the precision rate reaches 1, the recall rate reaches 0.895, and the area under the curve (AUC) finally stabilizes at 0.959.



(a) Precision rate (b) Recall rate

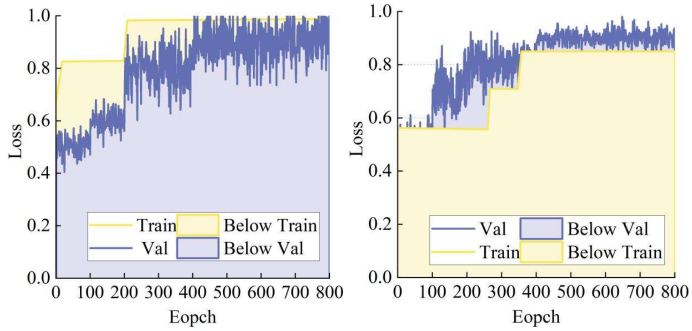


Figure 4: Training process

Table 5: Component score coefficient matrix

Index	Ingredient							
	F1	F2	F3	F4	F5	F6	F7	F8
(A11)	-0.773	0.73	-0.109	0.48	0.259	0.93	0.406	0.22
(A12)	0.095	-0.818	0.152	-0.913	0.487	-0.468	-0.423	-0.787
(A13)	-0.613	-0.443	-0.581	-0.782	0.151	0.324	0.558	-0.607
(A14)	0.379	-0.465	0.597	0.49	0.769	0.596	-0.246	0.055
(A21)	-0.066	0.161	-0.147	-0.843	-0.92	-0.703	0.165	0.478
(A22)	-0.51	0.562	-0.835	-0.734	0.974	-0.646	-0.441	-0.838
(A23)	-0.159	-0.061	0.379	-0.987	-0.206	0.626	-0.123	0.979
(A24)	-0.283	0.349	-0.142	0.724	-0.958	0.592	0.469	-0.968
(A25)	0.603	-0.422	-0.791	-0.739	0.857	0.46	0.231	0.097
(A31)	0.842	-0.093	-0.622	0.081	0.961	-0.315	0.334	-0.166
(A32)	-0.218	0.405	0.467	0.878	0.142	-0.37	0.866	0.281
(A33)	0.705	0.704	0.549	0.874	-0.478	0.862	-0.858	-0.748
(A34)	0.333	-0.939	-0.509	0.16	-0.393	-0.613	0.794	0.058
(A41)	0.39	0.942	-0.119	-0.562	-0.547	0.9	0.889	-0.615
(A42)	0.213	0.226	-0.885	0.243	-0.285	0.357	0.927	-0.823
(A43)	-0.412	-0.597	0.141	-0.338	0.617	0.919	-0.906	0.404
(A44)	-0.315	-0.164	-0.133	-0.952	0.787	0.464	0.894	-0.252
(B11)	0.807	-0.299	-0.494	0.717	0.505	-0.853	0.288	0.467
(B12)	0.786	-0.346	-0.686	0.825	0.914	0.534	0.62	0.486
(B13)	-0.626	-0.056	0.77	0.203	-0.715	-0.651	-0.482	0.226
(B21)	0.402	0.481	0.625	0.54	0.924	0.32	0.827	-0.845
(B22)	0.434	-0.497	0.884	-0.534	0.516	0.223	-0.231	0.718
(B32)	-0.057	-0.03	-0.624	0.799	-0.699	-0.377	0.809	-0.168
(B52)	0.558	-0.474	-0.592	-0.878	0.791	0.197	-0.688	-0.597
(B53)	-0.969	-0.812	0.271	0.776	-0.594	-0.805	-0.408	0.138
(B61)	0.744	-0.496	0.122	-0.372	-0.734	0.439	-0.407	-0.903
(B62)	-0.988	0.984	0.083	0.073	0.027	0.866	0.129	0.173

V. A. 2) Variable importance analysis

In this paper, Logisic model was chosen to analyze the importance degree of each factor variable. With financial status as Y and each early warning factor variable as X into the Logistic model, the coefficients of each factor variable determine the direction of the role of the independent variable on the dependent variable and the degree of influence, which is summarized according to the category and the degree of influence of the order. The degree of importance of each factor variable is shown in Fig. 5, among which, the importance degree of four factors, namely, (F2) profitability, (F3) development ability, (F4) operation ability and cash flow, and (F7) goodwill, is 0.800 and above, indicating that these four variables are important influences on the changes of corporate financial risk.

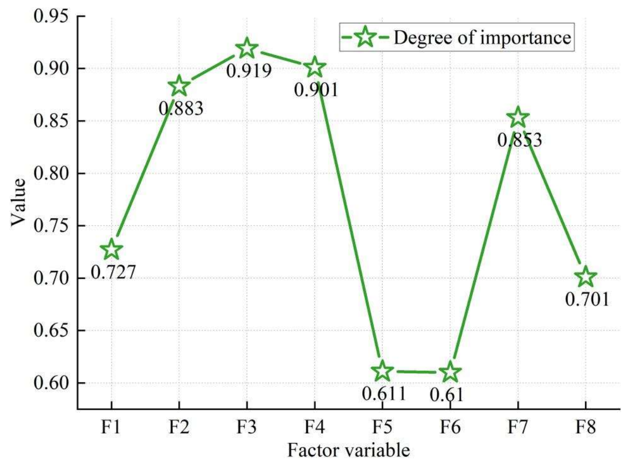


Figure 5: The importance of factor variables

V. B. Early warning performance of the model

In this section, two traditional corporate financial risk early warning models (C1 and C2) are selected as comparative modeling approaches to unfold the early warning performance performance assessment with the modeling approaches in this paper. The results of (I1) coefficient of determination and (I2) root-mean-square error of the three early warning modeling methods for the prediction of financial fraud risk of enterprise P in 2017-2021 are recorded separately in Table 6.The coefficients of determination of the prediction results of the three early warning methods show a yearly upward trend, and the root-mean-square errors show a yearly downward trend, which indicates that with the continuous increase in the enterprise's financial data, the three early warning methods are able to obtain effective data This indicates that with the increasing financial data of enterprises, all three early warning methods can obtain effective data basis, thus enhancing the performance of early warning. However, by comparing the coefficient of determination and root mean square error of the three early warning methods, it can be seen that the coefficient of determination of the experimental group's early warning method is always the largest, and the root mean square error is always the smallest, in which the coefficient of determination is greater than 0.900, and the root mean square error is less than 0.200. The above experimental results can prove that this paper's early warning modeling method has higher prediction accuracy for the risk of financial fraud, and stronger prediction reliability, which helps enterprises to improve financial management and enhance the reliability of prediction. It helps enterprises to improve financial management efficiency and avoid financial crisis.

Table 6: The prediction results of the risk of enterprise financial fraud

Year	Textual model		C1 Model		C2 Model	
	I1	I2	I1	I2	I1	I2
2017	0.932	0.186	0.776	0.332	0.882	0.377
2018	0.942	0.177	0.786	0.318	0.883	0.366
2019	0.952	0.168	0.797	0.302	0.884	0.342
2020	0.961	0.162	0.809	0.277	0.886	0.319
2021	0.972	0.154	0.816	0.241	0.888	0.302

V. C. Methods for improving the comprehensive efficiency of enterprises from the perspective of financial management

Based on the above analysis, this section gives the following four suggestions for the improvement of the comprehensive efficiency of enterprises from the perspective of financial management.

(1) Strengthen the importance of financial management work

As an important work involving capital operation, cost control and risk prevention, financial management has an important impact on the strategic decision-making and sustainable development of enterprises. Therefore, strengthening the importance of financial management at the strategic level is a key step to enhance the comprehensive competitiveness of enterprises. Strengthening financial management can promote the overall long-term healthy development of the enterprise, and provide a background for the enhancement of the comprehensive efficiency of the enterprise.

(2) Establish a sound financial internal control system

Enterprises need to establish a scientific concept of financial management, according to their own situation to develop appropriate internal control system, construction and strengthening of budget management, cost control and risk warning mechanism. Through quantitative management, to promote the operation of the economic activities of the enterprise with scientific and measurement, to promote the comprehensive efficiency of the enterprise.

(3) Improve the professional level and overall quality of financial management staff

The level of financial management work is closely related to the professionalism of the financial management team, therefore, enterprises should pay attention to the construction and optimization of the financial organizational structure. By strengthening the training of existing staff, regular education and training activities to improve the professional ability of the financial team to establish a professional financial management team. Thus ensuring that enterprises can carry out financial management work in colleges and universities, improve the efficiency of financial management, and guarantee the stability of the comprehensive efficiency of the enterprise to move forward and develop.

(4) Improve the intelligent level of financial management work

Enterprises should make use of big data, artificial intelligence and other digital technologies to carry out real-time monitoring and analysis of capital flow, information flow and business flow, and promote the improvement of informationization and intelligent development level of financial management. Ensure that the financial management work is carried out efficiently, improve the efficiency of financial management, and assist in the enhancement of the comprehensive efficiency of the enterprise.

VI. Conclusion

In the assessment of corporate financial risk, this paper establishes the early warning indicators of corporate financial risk from the financial and non-financial perspectives, which contains 9 evaluation dimensions and 27 third-level evaluation indicators, including debt service ability, profitability, development ability, operating ability and cash flow, equity structure, litigation and arbitration, goodwill, and equity pledge.

In the early warning of corporate financial risk, this paper proposes to improve the model prediction accuracy with the higher-order topology based on heterogeneous graph representation learning under the framework of topology optimization theory. Combined with the improved BAS-BP model, the enterprise financial risk early warning model is constructed. After training, the precision rate of the enterprise financial risk early warning model reaches 1, and the recall rate reaches 0.895. In the application of enterprise financial risk early warning, the coefficients of determination of the early warning results for five consecutive years for enterprise P are all greater than 0.900, and the root mean square errors are all less than 0.200.

In the improvement of the comprehensive efficiency of the enterprise, this paper, from the perspective of financial management, gives: (1) strengthen the importance of financial management work, (2) establish a sound financial internal control system, (3) improve the professional level and comprehensive quality of financial management staff, (4) improve the level of financial management work of the intelligence of the four strategic recommendations.

Acknowledgements

This work was supported by 2022 Scientific Research Preparation Plan Project of Anhui Province (Project approval number: 2022AH051850) and 2024 Anhui Provincial University Scientific Research Program (Project approval number: 2024AH052540; 2024AH052530).

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