International Journal for Housing Science and Its Applications
Publish August 3, 2025. Volume 46, Issue 3 Pages 1188-1201

https://doi.org/10.70517/ijhsa46392

A Study on Multi-Constraint Portfolio Optimization Based on Lagrange Multiplier Method in Financial Market Risk Management Framework

Mengrui Bao[1,*]
[1] Economics and Management Faculty, Cangzhou Normal University, Cangzhou, Hebei, 061000, China
Corresponding authors: (e-mail: baomengrui666@163.com).

Abstract With the improvement of economic development and consumption level, the portfolio problem has become a concern of more and more people. Under the financial market risk management framework, this topic is based on the classical portfolio MV model, and the generalized MV model with multi-conditional constraints is established by introducing a variety of trading constraints existing in the real trading, which is solved by using the Lagrange multiplier method. Collecting trading data from the Chinese securities market for empirical study of the model, it is found that the portfolio returns of securities in industries with small correlation are higher than those of securities in industries with large correlation, and the risk-to-investment ratios of securities in different industries and the same industry are 0.834-1.057 and 0.823-1.038 in the multi-group test, and diversification of investment in different industries can reduce investment risk. The model in this paper performs better in all indicators of portfolio performance, its cumulative return is higher than the comparison method by 0.118~0.213, and the yield curve is stable at -0.024~0.025.The results show that the proposed multi-constraint portfolio model is not only reasonable and effective, but also can better guide investors to choose the optimal and robust investment program.

Index Terms Lagrange multiplier method, MV model, multiconditional constraints, portfolio, financial market

I. Introduction

With the continuous advancement of economic globalization, the development of financial markets is extremely rapid, although the financial market provides convenience for the flow of capital breaks, but there are also many risks [1], [2]. In order to effectively control the risk, financial market risk, management has become an increasingly popular topic [3]. At the same time, multi-constrained portfolio optimization is also an essential part of the financial market [4], [5].

Financial market risk management refers to a series of activities to monitor, identify, assess, control and transfer potential, actual and unknown risks in financial markets [6], [7]. Financial market risk is mainly categorized into market risk, credit risk, operational risk, liquidity risk, legal risk and so on [8]. However, simply understanding financial market risk is not enough. In order to effectively manage risks, it is necessary to improve risk identification and analysis capabilities, and actively explore the use of various tools and methods for risk management [9]-[11]. At the same time, enterprises must establish implementation rules for risk management based on factors such as enterprise type, operation strategy and financial objectives, and carry out moderate and efficient risk management [12], [13].

Multi-constrained portfolios, on the other hand, combine different classes of assets to achieve the purpose of reducing the risk of a single investment and increasing the overall return [14], [15]. On the one hand, it is for optimized risk and return, and on the other hand, it is for precise control of risk and optimal return, and it reduces the risk of a single investment through diversification [16]-[18]. The volatility of the portfolio can be minimized by selecting assets of different types, different sectors, and different prices for the portfolio [19], [20]. Also, placing stocks of different companies in the same industry in the same portfolio can reduce unsystematic risk [21], [22]. Its another purpose is to help investors to get the highest return, and investors can maximize the maximum benefit by investing rationally in each asset [23]-[25].

Aiming at the shortcomings of the classical mean-variance model in which the assumptions are too harsh and at the same time not applicable to real life, the study considers various types of investment constraints existing in real economic activities, constructs a generalized MV model with multiple conditional constraints, and adopts the Lagrange multiplier method to solve the optimization problem. Publicly traded data of several Chinese securities markets from 2020 to 2024 are collected as research samples, and 50 securities in different industries and the same industry are selected for simulation analysis, comparing the risk-to-investment ratios and the optimal solution results of five different portfolio models to test the application effectiveness and rationality of the proposed generalized MV model. Then, using yield evaluation, risk evaluation and annualized risk return as the evaluation indexes of portfolio performance, we compare this paper's model with the other three methods and analyze the yield curves and cumulative yield curves of the four methods to investigate the return-return and risk-prevention performance of the proposed multi-constrained portfolio model.

II. Portfolio modeling under multiple constraints

With social progress and economic development, increased disposable income, and increased awareness of financial management, the modern financial market provides a variety of products for investors to choose from in order to meet people's investment needs, including stocks, bonds, futures, and so on. There are two decision-making model frameworks in portfolio optimization theory, including utility maximization theory and return-risk trade-off theory. In the return-risk trade-off theory, risk can be specified as a number. In this paper, the classical mean-variance model (MV model) portfolio is optimized with multiconditional constraints under the financial market risk management framework and solved by using the Lagrange multiplier method to propose a portfolio model under multiconditional constraints.

II. A.Mean-variance model

In the mean-variance (M-V) theoretical model, the expected value of the rate of return, i.e., the average return on assets, is taken as the investment return, while the fluctuation of the rate of return, i.e., the variance of the return on assets, is taken as the investment risk. The basic idea of this model is: in the rational behavior of investors, “avoidance of investment risk”, “non-satisfaction of return” and other conditions are met, to solve the optimal asset allocation ratio coefficients, and ultimately to achieve a certain level of return under the constraints of risk Minimize the risk under the constraint of a certain level of return, or maximize the return under the constraint of a certain level of risk. It can be seen that the theory is mainly based on the assumption of rational market, and its basic assumption is that the investor decides the portfolio strategy for a fixed period of time based on the probability distribution of the return of the financial assets, i.e., the proportion of the total assets allocated to each investment species. The classical mean-variance model does not take into account the impact of real-world factors such as trading costs, volume limits, trading spreads and personal income tax charges on asset allocation strategies, i.e., it assumes that there are no frictions in the market and that the set of market information known to all investors in the market is the same.

The model also demonstrates that if all of its assumptions are satisfied, the set of points of all possible mean-variance combinations that could serve as an investor's optimal portfolio strategy is in fact a parabola on a two-dimensional plane. The investor's goal is to pursue the optimal mean-variance portfolio, then he or she is looking for the mean-variance portfolio that best matches his or her own risk preferences on this curve. In other words, the basis for an investor's choice of different portfolios on the mean-variance model's efficient frontier (efficient curve) lies in his or her own different preferences or aversions to risk. Therefore, it can be learned that: if a certain two investors have the same risk preference or risk aversion, then they carry out financial investment transactions (on the mean-variance of the effective portfolio selection) of the non-differentiated curve is parallel to each other, and investors seek the optimal portfolio is their own investment transactions when the non-differentiated curve and the mean-variance of the effective frontier (effective curve) of the point of tangency.

Assume that R_i denotes the rate of return (random variable) on the i th trading asset invested in, with mean $r_i = E(R_t)$, and covariance matrix of $G = (\sigma_{ij})_{n\times n}\ i, j = 1,2,...,n$. $\sigma_{ij} = COV(R_i, R_j) = E[(R_i - E(R_i))(R_j - E(R_j))]$, x_i denotes the proportion of the capitalization invested in the first i secured asset $i, j = 1,2,...,n$,. And remember that $R = (r_1, r_2, ...r_n)^T$, $X = (x_1, x_2, ...x_n)^T$, e denotes a unit vector. Since X denotes the asset allocation vector of the portfolio, it must satisfy the following conditions:

$$\sum_{i=1}^{n} x_i = 1 \tag{1}$$

Or:

$$e^T X = 1 \tag{2}$$

The expected return and variance of the portfolio are shown in the following equations, respectively:

$$r_p = X^T R \tag{3}$$

and:

$$\sigma_p^2 = X^T G X \tag{4}$$

The classical mean-variance portfolio model can be expressed as:

$$\min X^T GX / 2$$
$$s.t \begin{cases} X^T R = r_0 \\ e^T X = 1 \\ x_i \geq 0 \end{cases} \tag{5}$$

Or:

$$\max r(x) = X^T R$$
$$s.t \begin{pmatrix} X^T GX = \sigma_0^2 \\ e^T X = 1 \\ x_i \geq 0 \end{pmatrix} \tag{6}$$

II. B.MV equations under multiconditional constraints

Based on the above research, this paper considers more systematically the various constraints existing in real trading, and establishes a generalized MV model with multiple investment constraints by multi-faceted generalization of the traditional MV model.

II. B. 1) Constraints

(1) Investment budget constraint

Let $p = (p_1, p_2, \cdots, p_n)^T$ denote the vector of prices for n securities, and $x = (x_1, x_2, \cdots, x_n)^T$ denote the investment in the n security's trading volume vector, and b denotes the maximum amount of money that the investor can use to invest, this constraint can be expressed as $p^T x \leq b$.

(2) Trading volume constraint

Use l_i to denote the limit on the number of trades in a security set by the investor or imposed by the securities market. This constraint can be expressed as $l_i \leq x_i \leq u_i$, if $l_i < 0$ means that the investor buys the security i, then the purchase volume should be located in the interval $[0, u_i]$, and if the sale then the sale volume should be in the range $[l_i, 0]$.

(3) Minimum trading unit constraint

The minimum buy transaction volume in the securities market is 100 shares and the minimum sell transaction volume is 1 share. This trading constraint can be expressed as $\frac{x^b}{100} \in Z^+, x^s \in Z^+$, where x^b, x^s denotes the quantity of i th security purchased and sold respectively.

(4) Industry (sector investment ratio) constraints

Assuming that there are a total of m categories of industries, and the investor wishes to invest in the j th category of industries with the upper and lower investment ratios of $a_j \leq X_j \leq \overline{a}_j, j = 1, \cdots, m$, the constraint can be expressed as follows: $a_j \leq \sum_i^m x_i D_j \leq \overline{a}_j, j = 1, \cdots, m$. where D_j is a dummy variable that takes the value of 1 when stock i belongs to the j th class of industry (sector), and 0 otherwise.

(5) Constraint on the size of the total portfolio change

The constraint can be described as $\sum_{i=1}^{n} (x_i - x_i^0) \leq C$, where C is the total amount of purchases set by the investor, and x_i^0, x_i denotes the amount of holdings of i securities before and after the portfolio, respectively.

(6) Transaction cost constraints

Transaction costs mainly include transaction fees, transfer fees and stamp duty, etc. These costs are charged as a percentage of the turnover, assuming that the proportional cost coefficients of buying and selling are Q_b, Q_i, these costs can be expressed as: $Q_b p_i (x_i - x_i^0)^+, Q_b p_i (x_i - x_i^0)^-$, which is $(x_i - x_i^0)^+ = \max(x_i - x_i^0)$, $Q_b p_i (x_i - x_i^0)^- = -\min Q_b p_i (x_i - x_i^0)$.

(7) Minimum rate of return constraint

When taxes and transaction costs are considered, the expected net return function $R(x,y)$ is: $R(x,y) = y^T x - t_1 - t_2 - q^T x$.

Then there is at a minimum rate of return of μ:

$$R(x,y) = y^T x - t_1 - t_2 - q^T x(1+k) \geq \mu \left\{ \sum_{i=1}^{n} q_i x_i + \sum_{i=1}^{n} k_i q_i (u_i + v_i) \right\} \tag{7}$$

Substituting t_1, t_2, we have:

$$\begin{aligned} R(x,y) &= \sum_{i=1}^{n} y_i x_i - \sum_{i=1}^{n} k_i q_i (u_i + v_i) \\ &- \sum_{i=1}^{n} k y_i x_i - q^T x^o (1+k) \\ &\geq \mu \left\{ \sum_{i=1}^{n} q_i x_i + \sum_{i=1}^{n} k_i q_i (u_i + v_i) \right\} \end{aligned} \tag{8}$$

where $x_i - x_i^0 = u_i - v_i, u_i \geq 0, v_i \geq 0$.

The above constraints can be expressed in terms of 0-1 variables, let $y_i \in \{0,1\}, y_i = 1$ denote investing in security i, and the corresponding $y_i = 0$ denote not investing in security i, and each of the three types of constraints mentioned above can be expressed as follows:

Dependent investment constraint: $y_i \geq y_j, i \in N_1, j \in N_2$.

Associative investment constraint: $y_i \leq y_j, i \in N_3, j \in N_4$.

Repulsive investment constraint: $y_i \leq 1 - y_j, i \in N_5, j \in N_6$.

II. B. 2) Generalized MV modeling

In order to obtain a concise and uniform mathematical expression of the multiconditional constraint, four variables $x_i^b, x_i^s, z_i^b, z_i^s$ are introduced. In the minimum trading unit constraint, x_i^b denotes the buying quantity of security i and satisfies $x_i^b \geq 0$, x_i^s denotes the selling quantity of security i and satisfies $x_i^s \geq 1$, and z_i^b, z_i^s is a 0-1 variable, when $z_i^b = 1$ indicates a real purchase of the security i, $z_i^s = 1$ indicates a real sale of the security i, and $z_i^b = z_i^s = 0$ indicates neither purchase nor sale. Since the holder cannot perform both buying and selling operations, $z_i^b + z_i^s \leq 1$ should be satisfied. The investor's final holding of security i is $x_i = x_i^0 + x_i^b - x_i^s$.

If we remember that $X^b = (x_1^b, \cdots, x_n^b)^T, X^s = (x_1^s, \cdots, x_n^s)^T$, the above constraints can be simplified anew as:

Budget constraint: $p^T (X^b - X^s) \leq b$.

Transaction costs: $Q_b p_i x_i^b, Q_s p_i x_i^s$.

The total portfolio change size constraint: $\sum_{i=1}^{n} (x_i^b - x_i^s) \leq c$.

The logical constraint can be formulated as:

$$\begin{aligned} & z_i^b + z_i^s \geq z_j^b + z_j^s, i \in N_1, j \in N_2 \\ & z_i^b + z_i^s \leq z_i^b + z_i^s, i \in N_3, j \in N_4 \\ & z_i^b + z_i^s \leq 1 - (z_j^b + z_i^s), i \in N_5, j \in N_6 \\ & z_i^b + z_i^s \leq 1; z_i^b, z_i^s \in 0,1, i = 0,1,\cdots,n \end{aligned} \tag{9}$$

Volume limit constraint: $z_i^b u_i^0 \leq x_i^b \leq z_i^b u_i, z_i^s l_i \leq x_i^s \leq z_i^s l_i^0$, where u_i, u_i^0 denote the upper and lower bounds for purchasing the ith security, respectively, while l_i, l_i^0 denote the upper and lower bounds for selling the ith security.

Integrating the MV equation under the multi-reality condition constraint and the CVaR model risk assessment method constraint, this equation is modeled as follows:

$$\begin{aligned}
&\min\frac{1}{2}x^T Vx-\alpha\sum_{i=1}^{n}r_i p_i(x_i^0+x_i^b-x_i^s)+Q_b\sum_{i=1}^{n}p_i x_i^b+Q_i\sum_{i=1}^{n}p_i x_i^i\\
&s.t. A(X^b-X^s)\le b,\sum_{i\in N_i}(x_i^b-x_i^s)\le C\\
&z_i^b u_i^0\le x_i^b\le z_i^b u_i, i=1,2,\cdots,n\\
&z_i^b+z_i^s\ge z_j^b+z_j^s, i\in N_1, j\in N_2\\
&z_i^b+z_i^s\le z_j^b+z_i^s, i\in N_3, j\in N_4\\
&z_i^b+z_i^s\le 1-(z_j^b+z_j^s), i\in N_5, j\in N_6\\
&z_i^b+z_i^s\le 1; z_i^b, i=1,2,\cdots,n, z_i^b, z_i^s\in\{0,1\}, i=1,2,\cdots,n\\
&x_i^b, x_i^s\in Z^+, i=1,2,\cdots,n\\
&CVaR_k=VaR_k+E[f(x,y)-VaR_k\mid f(x,y)>VaR_k]\\
&=(1-k)^{-1}\int_{f(x,y)\ge VaR(x)}f(x,y)p(y)dy
\end{aligned}\tag{10}$$

where A is a matrix of investment constraint coefficients consisting of a linear combination of budget constraint coefficients and other available investment quantities.

II. C.Lagrangian optimization method

Portfolio investment decision problems can often be abstracted as continuous dynamic optimization problems. In solving continuous dynamic optimization problems, Pontryagin's maximum principle (which in essence applies the Lagrange multiplier method) or the dynamic programming method can be used if it is a deterministic model, and the Lagrange multiplier method is generally not applicable if it is a stochastic model. In this paper, the Lagrange multiplier method is utilized to solve the multiconstrained portfolio model.

Let $x(t)$ and $u(t)$ be the $p\times 1$ vectors of the state variables and the $q\times 1$ vectors of the control variables at the moment of t, respectively. If clearly understood, the independent variable t is suppressed.

Assume that the stochastic model is:

$$dx=f(x,u)dt+S(x,u)dz\tag{11}$$

where $dx(t)=x(t+dt)-x(t)$, $z(t)$ is an $n\times 1$ vector Wiener process with covariance matrix $\mathrm{cov}(dz)=\Phi dt$, and S is a $p\times n$ matrix. The Σdt will denote the covariance matrix $cov(Sdz)=S\Phi S'$. Let $r(x,u)$ be the rate of return or utilization of the cash flow with the objective of maximizing the expected value:

$$E\int_0^{\infty}e^{-\beta t}r(x,u)dt\tag{12}$$

In order to solve the problem using the Lagrangian method, the components $\lambda(x)$ of the p vector of the Lagrangian multipliers are utilized to form the Lagrangian expression shown below, based on the constraints of the objective function (12) and the stochastic differential equation (11):

$$\begin{aligned}
L=&\int_0^{\infty}E_t\{e^{-\beta t}r(x,u)dt-e^{-\beta(t+dt)}\lambda'(t+dt)\\
&\times[x(t+dt)-x(t)-f(x,u)dt-S(x,u)dz]\}
\end{aligned}\tag{13}$$

Among other things, the conditional expectation E_n can be proved to be reasonable by the following statement of the problem: that is, when determining the $u(t)$ of the control variable, given that the t moments contain information about $x(t)$, which has changed the order of the integration, taking the stochastic integral $\int gdz$ has been proved to be a reasonable expectation, as defined by the Ito of the stochastic function g. Setting it to 0, the derivatives of L with respect to $u(t)$ and $x(t)$ will produce a set of first-order conditions for optimization if the Ito differentiation rule is applied to evaluate the vector $d\lambda$. The i th component of $d\lambda$ is:

$$d\lambda_i = \left[\frac{\partial \lambda_i}{\partial x'} f + \frac{1}{2}\mathrm{tr}\left(\frac{\partial^2 \lambda_i}{\partial x \partial x'}\Sigma\right)\right]dt + \frac{\partial \lambda_i}{\partial x} S(x,u)dz, i = 1,\ldots,p \tag{14}$$

where $\partial\lambda_i / \partial x'$ is the $1\times p$ vector and $\partial^2\lambda_i / \partial x\partial x'$ is the $p\times p$ matrix. In Eq. (14), the function λ_i is assumed to be in a steady state, then it is not bounded by t. Finding the derivative of Eq. (13) with respect to $ui(t)$ yields:

$$\begin{aligned} e^{\beta t}\frac{\partial L}{\partial u_i} &= \frac{\partial r}{\partial u_i}dt + e^{-\beta dt}\left[\frac{\partial f'}{\partial u_i}E_t\lambda(t+dt)dt + E_t dz'\frac{\partial S'}{\partial u_i}\lambda(t+dt)\right] \\ &= \frac{\partial r}{\partial u_i}dt + (1-\beta dt)\left[\frac{\partial f'}{\partial u_i}(\lambda dt + E_t d\lambda dt) + E_t dz'\frac{\partial S'}{\partial u_i}(\lambda + d\lambda)\right] + o(dt) \\ &= \frac{\partial r}{\partial u_i}dt + \frac{\partial f'}{\partial u_i}\lambda dt + (1-\beta dt)E_t tr\left[\frac{\partial S'}{\partial u_i}d\lambda dz'\right] + o(dt) \\ &= \frac{\partial r}{\partial u_i}dt + \frac{\partial f'}{\partial u_i}\lambda dt + tr\left[\frac{\partial S'}{\partial u_i}\frac{\partial \lambda}{\partial x'}S\Phi\right]dt + o(dt) \\ &= 0, i = 1,\ldots,q \end{aligned} \tag{15}$$

where the order of any product of $dt, d\lambda$ and dz (the order is $\sqrt{dt}$) is less than dt, $E_t dz = 0$, and substituting Eq. (14) for $d\lambda, \partial\lambda / \partial x'$ is the $p\times p$ -matrix. $\lambda_i(t+dt)[x_i(t+dt) - \cdots - S_i' dz]$ denotes the i th component of the inner product $\lambda'(t+dt)[x(t+dt)\cdots]$, where S_i' denotes the i th row of S, obtained:

$$\begin{aligned} e^{\beta t}\frac{\partial L}{\partial x_i} &= \frac{\partial r}{\partial x_i}dt - \lambda_i + e^{-\beta dt}\left[E_t\lambda_i(t+dt) + \frac{\partial f'}{\partial x_i}E_t\lambda(t+dt)dt + E_t dz'\frac{\partial S'}{\partial x_i}[\lambda(t+dt)]\right] \\ &= \frac{\partial r}{\partial x_i}dt - \lambda_i + (1-\beta dt)\left[\lambda_i + E_t d\lambda_i + \frac{\partial f'}{\partial x_i}\lambda dt + E_t dz'\frac{\partial S'}{\partial x_i}(\lambda + d\lambda)\right] + o(dt) \\ &= \frac{\partial r}{\partial x_i}dt - \beta\lambda_i dt + \left[\frac{\partial \lambda_i}{\partial x}f + \frac{1}{2}tr\left(\frac{\partial^2\lambda_i}{\partial x\partial x'}\Sigma\right)\right]dt + \frac{\partial f}{\partial x_i}\lambda dt + tr\left[\frac{\partial S}{\partial x_i}\frac{\partial \lambda}{\partial x}S\Phi\right] \\ & dt + o(dt) = 0, i = 1,\ldots,p \end{aligned} \tag{16}$$

Eqs. (15) and (16) are two first-order conditions for the optimal control u and the Lagrange multiplier 1. To ensure that the solutions to Eqs. (15) and (16) are maximal, the second-order conditions for the Lagrange multiplier method must be tested. If it is a non-stochastic model, both S and $\Sigma = S\Phi S'$ are 0. All trace terms in the last line of Eqs. (15) and (16) vanish. The solution will degenerate to a solution in continuous time corresponding to a non-stochastic optimal control problem with a dynamical system given by the following equation:

$$dx = f(x,u)dt \tag{17}$$

In the above solution, it is assumed that λ is only a function of x and not of t. If the assumption is relaxed, the term $\partial\lambda_i / \partial t'$ will appear inside the square brackets of Eq. (14) for $d\lambda_i$. And the same term will appear inside the square brackets after the last equal sign of Eq. (16) to multiply with dt and produce the partial differential equation of λ as follows:

$$\frac{\partial\lambda}{\partial t} + \frac{\partial\lambda}{\partial x'}f + \frac{\partial f'}{\partial x}\lambda + \frac{\partial r}{\partial x} - \beta\lambda = 0 \tag{18}$$

Eq. (17) and Eq. (15) with trace terms omitted, or:

$$\frac{\partial r}{\partial u} + \frac{\partial f'}{\partial u}\lambda = 0 \tag{19}$$

Provide a pair of equations to u and λ. These equations can be derived from the well-known Pontryagin's maximum principle for solving nonstochastic optimal control problems in continuous time. In order to apply Pontryagin's maximum principle to solve non-stochastic control problems, Hamiltonian functions are constructed:

$$H = r(x,u) + e^{-\beta dt}\lambda' f(x,u) \tag{20}$$

And the setup:

$$\frac{\partial H}{\partial u} = 0 \tag{21}$$

$$\frac{\partial (e^{-\beta dt}\lambda)}{\partial t} = -\frac{\partial H}{\partial x} \tag{22}$$

$$\frac{\partial x}{\partial t} = \frac{\partial H}{\partial (e^{-\beta dt}\lambda)} \tag{23}$$

When Eq. (22) gives the differential equations for the state-variable dynamical system, Eqs. (20) and (21) are the same as the first-order conditions (18) and (17), respectively.

III. Empirical analysis

III. A. Data selection

This paper is based on January 1, 2020 to June 1, 2024, China's commodity futures market daily data as the object of study, excluding the futures types with fewer trading days or smaller turnover during the sample period, and obtaining a total of 23 futures varieties. The data are obtained from the public data of Dalian Commodity Exchange, Zhengzhou Commodity Exchange and Shanghai Futures Exchange. A specific multi-constraint portfolio model is constructed to conduct a simulation study on the selection of the optimal portfolio. The simulation study in this paper is carried out in two parts: one is to select 50 securities from different industries for the simulation study, and the other is to select 50 securities from the same industry for the study. In addition, the dataset is divided into a training set and a test set, in which the ratio of training data to test data division is 0.8:0.2, which is used as an analytical sample for portfolio performance evaluation.

III. B. Simulation analysis

III. B. 1) Situation in different industries

Five portfolio models were built by picking 50 companies from different industries that have been listed for a long time and are large in size in the following manner:

(1) Test 1: Solve the classic MV model, but here use x_i to denote the specific number of shares invested in the i th stock, and specify that the total trading volume is 1500, with an expected return greater than 250.

(2) Test 2: Solve the classical MV model, but require the solution to be integer, and the remaining parameter values are the same as Test 1.

(3) Test 3: Solve the generalized MV model, but do not consider the logistic constraints, the specific parameter values are: the initial holding of each security is 0, the risk preference factor is 1, and the proportional transaction cost coefficient is 0.7%. The specified trading volume is 1500, the maximum limit of funds that can be invested is 20000, the expected return is greater than 250, the minimum trading volume to buy each security is 0, the maximum trading volume is 10000, and the selling restriction is 0. The solution takes integer values.

(4) Test 4: differs from the previous test by the addition of logical investment constraints, as follows: the 5th and 6th securities are dependent investments, the 25th and 26th securities are repulsive investments, and the 45th and 46th securities are associative investments.

(5) Test 5: differs from Test 4 in containing more complex logical constraints, specifically: the 5th security and the 6th security, the 20th security and the 21st security are dependent investments, the 25th security and the 26th security, the 30th security and the 31st security are repulsive investments, and the 10th security and the 11th security, and the 45th security and the 46th security are associative investments.

Using the Lagrange multiplier algorithm to solve the above five models, in order to compare the different models in practice to find the investment program is good or bad, the definition of “risk to investment ratio = return / risk”, that is, to bear the unit of risk obtained by the return, the results of portfolio selection of different industries are shown in Table 1, the optimal results of portfolio selection The optimal solution is shown in Figure 1. The optimal portfolio found in test 1 selects 13 securities, at which time the risk-to-investment ratio is 1.057. In test 2, due to the addition of the integer value constraints, the risk increases, the return decreases, and the risk-to-investment ratio is

0.978, which is closer to test 1. Comparing the optimal solutions for Test 2 and Test 3 reveals that although the selected securities remain unchanged, there is a large change in the amount of individual securities purchased. The risk-to-investment ratio in Test 3 is 0.834, which is smaller than the risk-to-investment ratio in Test 2. This is due to the increase in risk and decrease in return caused by the increase in constraints.

Test 4 introduces logical constraints in the test, so after selecting the 25th security, the 26th security cannot be selected, and compared to the previous tests, Test 4 only selects 12 securities out of the previous 13 securities, and the risk of the investment increases further. Test 5 doubled the number of logical constraints, securities 26 and 30 were excluded from the optimal portfolio, and the new portfolio contained only 11 securities.

Tests 1 through 5 progressively increase the number of constraints, with a monotonically increasing trend in risk and a decreasing and then increasing expected return. In the last three tests, returns increased more than risk. So as the number of constraints increases, the risk-to-investment ratio also increases, which suggests that with the proper definition of the constraints in question, it is possible to invest in a smaller number of securities and still achieve a better investment result, thus avoiding the annoyance associated with full diversification. The risk-to-investment ratio in the classical MV model is higher than that in the generalized MV model because the classical MV model is constructed with too many assumptions and discards many realistic trading constraints. Therefore, although the risk-to-investment ratio in the classical MV model is high, it does not provide a real reference for actual investment operations.

Table 1: Portfolio selection results for different industries

Test number	Risk	Expected return	Income/risk
1	0.316	0.334	1.057
2	0.323	0.316	0.978
3	0.326	0.272	0.834
4	0.329	0.279	0.848
5	0.331	0.282	0.852

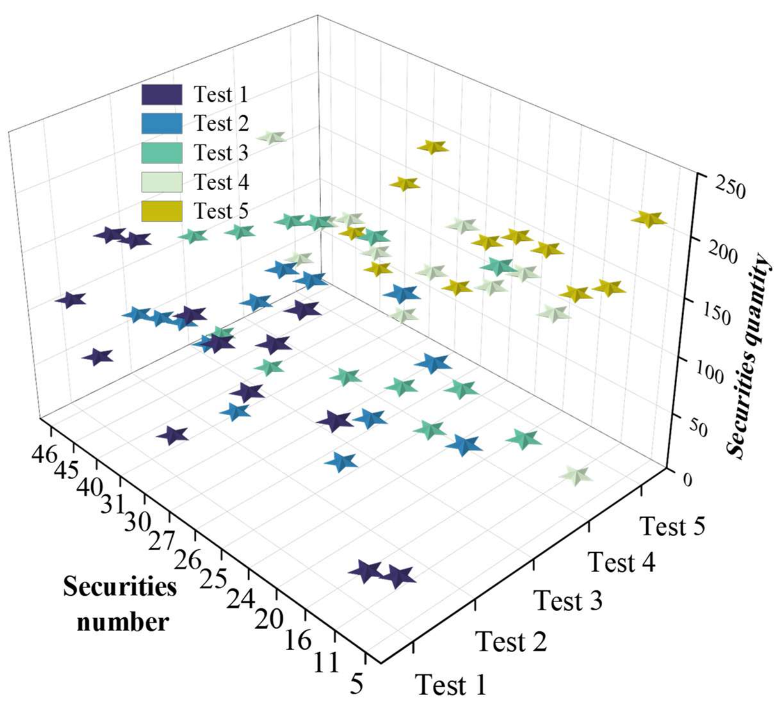

Figure 1: The optimal solution to the portfolio selection results

III. B. 2) Situation in the same industry

Selecting 50 securities from the same industry, the method of five modeling and the selection of each parameter value in different tests are also the same as in the previous group, the results of portfolio selection in the same industry are shown in Table 2, and the results of the optimal solutions of the five tests are shown in Figure 2. Comparison with the corresponding results in the first group reveals that after investing in the same industry, the portfolio risks are all greater than those in the first group, while the returns are all reduced, and thus the corresponding risk-to-investment ratios are all smaller than those in the first group, which are 1.038, 0.960, 0.823, 0.831, and 0.833, respectively.This suggests that when investing in the same industry, due to the high correlation between these securities, the risks cannot be effectively diversified and reduced. Risks cannot be effectively diversified and minimized, and higher risks must be taken in order to obtain the same returns. In these five tests, the trends of risk, expected return, and risk-to-investment ratio are exactly the same as the results in the previous group, in which 18 securities were selected and the amount of investment tends to average out over most of the selected securities.

Table 2: Portfolio selection results for the same industries

Test number	Risk	Expected return	Income/risk
1	0.320	0.332	1.038
2	0.326	0.313	0.960
3	0.328	0.270	0.823
4	0.332	0.276	0.831
5	0.335	0.279	0.833

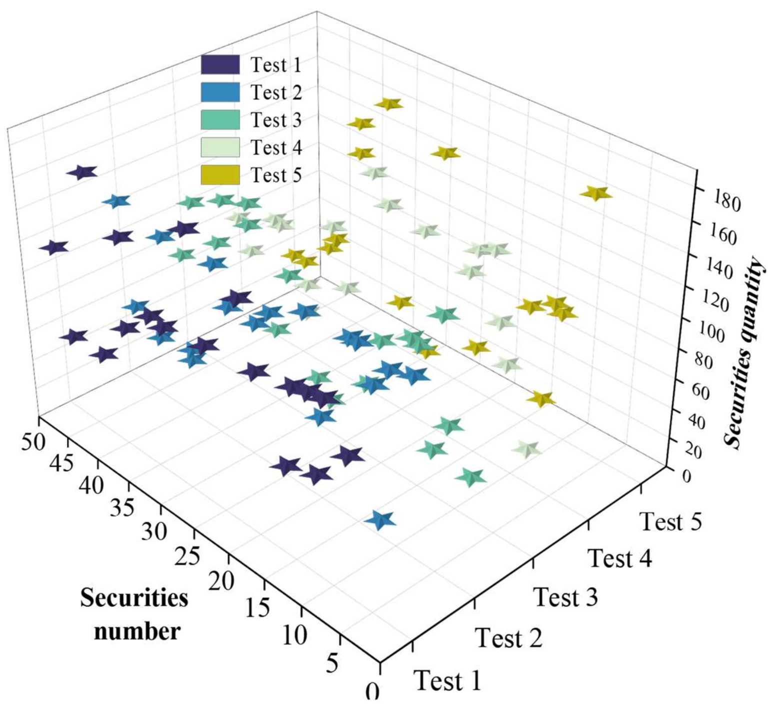

Figure 2: The optimal solution results of five tests

III. B. 3) Yield curve

Further analysis reveals that the securities with relatively large investments are companies with relatively stable earnings performance over the sample interval. Figure 3 shows the monthly yield volatility curve for the 27th security in the first group from January 2000 to December 2007. Figure 4 shows the monthly yield volatility curve for the 29th security in the second group from January 2000 to December 2007. The 27th security is more profitable in the sample interval and the volatility of the return is not very large, so choosing this type of security to invest in can get a better return and take less risk. And as can be seen from Figure 4, the performance of this security is quite stable in the later period, the yield mostly hovers above and below the value of 0. Investing in this kind of security is less risky, and choosing a certain amount of this kind of security in large-scale investment has a quite important role in controlling the risk effectively.

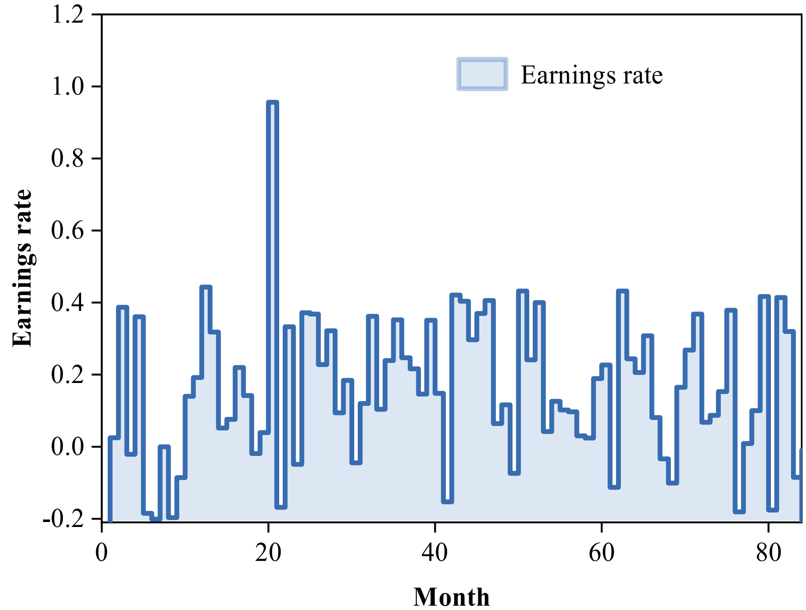

Figure 3: The monthly yield fluctuation curve of the 27th securities in the first group

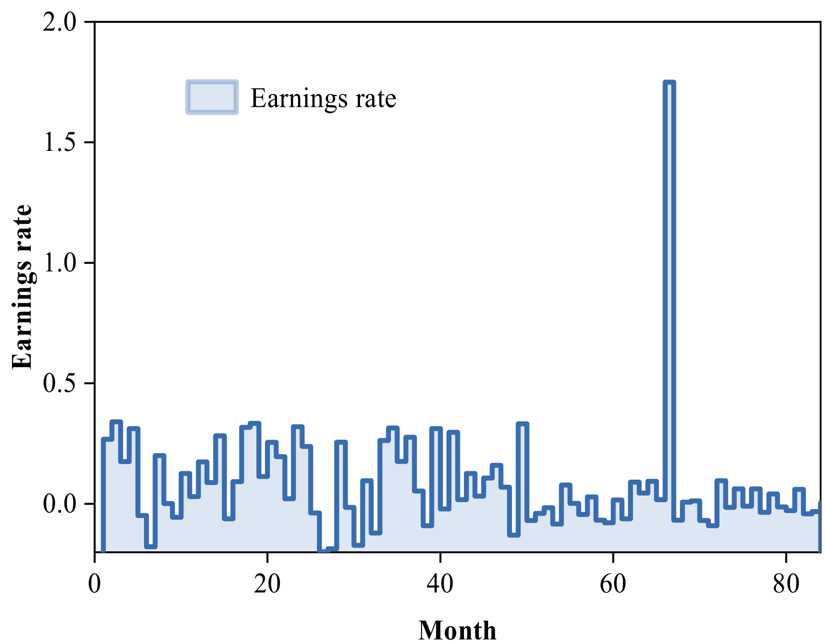

Figure 4: The monthly yield fluctuation curve of the 29th securities in the second group

III. C. Portfolio performance evaluation

III. C. 1) Evaluation indicators

In order to compare the performance of four investment strategies, namely MV model, LSTM+1/N model, LSTM+MV model and generalized MV model in this paper, three sets of performance evaluation indexes are introduced in this paper, which are return evaluation index, risk evaluation index and annualized risk return index. Specifically, the return evaluation indexes include Mean return, Standard deviation, Maximum, Minimum, Accumulated return, Risk evaluation indexes include VaR, Maximum-drawdown, Downside ratio, and Annualized risk-return indexes include Sharperatio, Sortino ratio, Omega ratio, Calmar ratio indicators.

III. C. 2) Rate of return evaluation

The returns of the four portfolios are first evaluated, and Figure 5 shows the evaluation of the portfolio performance returns. As can be seen from the standard deviation of the returns, the LSTM+1/N model has the largest fluctuation, with a Standard deviation of 0.0275, and the traditional MV model has the smallest fluctuation, with 0.0122. The maximum return of the LSTM+1/N model is much higher than that of the other two models, with a Maximum of 0.1025, which suggests that the model sets the investment proportion of the assets more aggressive. In terms of cumulative return, the generalized MV model in this paper is in the leading position, with Accumulated return reaching 0.2257, which is higher than the other three models by 0.0572, 0.0686 and 0.0288, respectively. In terms of return indicators, the portfolio of the traditional MV model is more conservative, and the portfolio of the generalized MV model under the multiconditional constraints in this paper not only produces higher returns, but also reduces the return volatility.

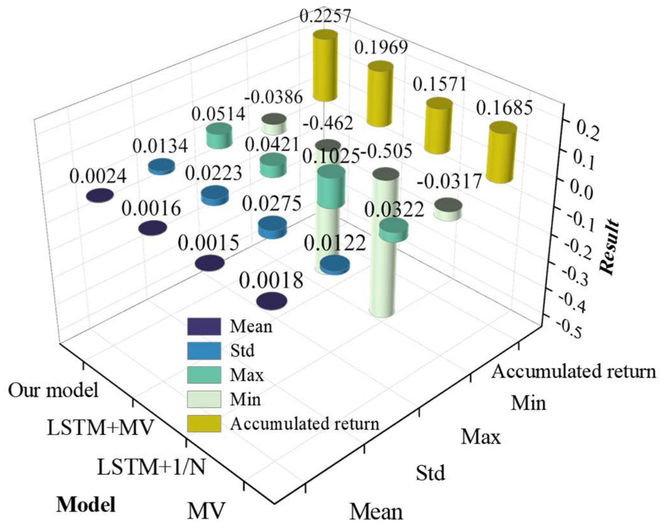

Figure 5: Evaluation of yield rate of portfolio

III. C. 3) Risk evaluation

Then the performance risk of the four portfolios is evaluated, and the portfolio performance risk evaluation is shown in Figure 6. It is proved that the generalized MV model under multiconditional constraints not only brings higher cumulative return, but also has outstanding performance in risk prevention, whether it is the maximum retracement index, downside risk index or 1-percentVaR and 5-percent VaR indexes show better risk prevention ability, and the risk evaluation of the generalized MV model under multiconditional constraints for each index is smaller than that of the LSTM +1/N and LSTM+MV models, which are -0.1052, 0.0318, 0.0275 and 0.0214, respectively.

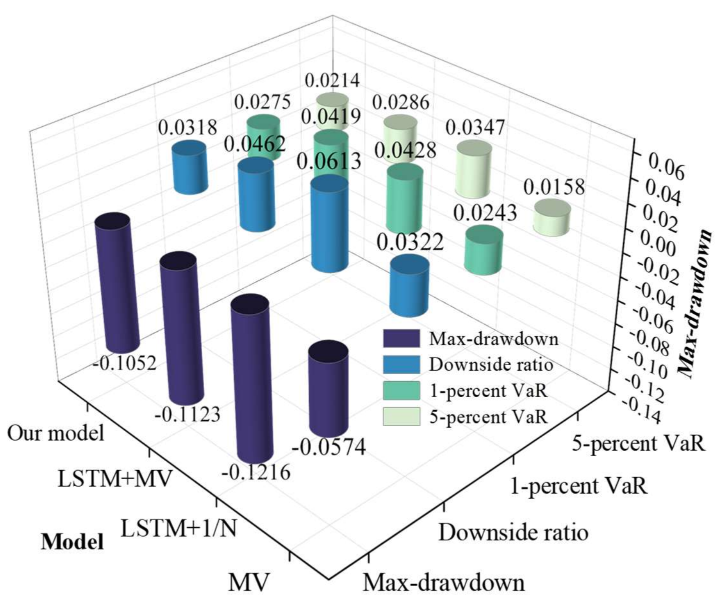

Figure 6: Performance risk evaluation of portfolio

III. C. 4) Risk-benefit evaluation

Finally, the three models are evaluated for their risk-adjusted returns. Figure 7 shows the risk-return evaluation of portfolio performance. The generalized MV model with multiconditional constraints has the best risk-return among the three strategies, and its Sharpe ratio, Sortino ratio, Omega ratio, and Calmar ratio are 0.0251, 0.4244, 1.0688, and 0.3552, respectively, which are higher than that of the three other The LSTM+1/N model is the worst performer, with a negative Sharpe ratio (-0.0421), indicating that taking one unit of risk not only does not result in excess payoff, but may also result in a loss.

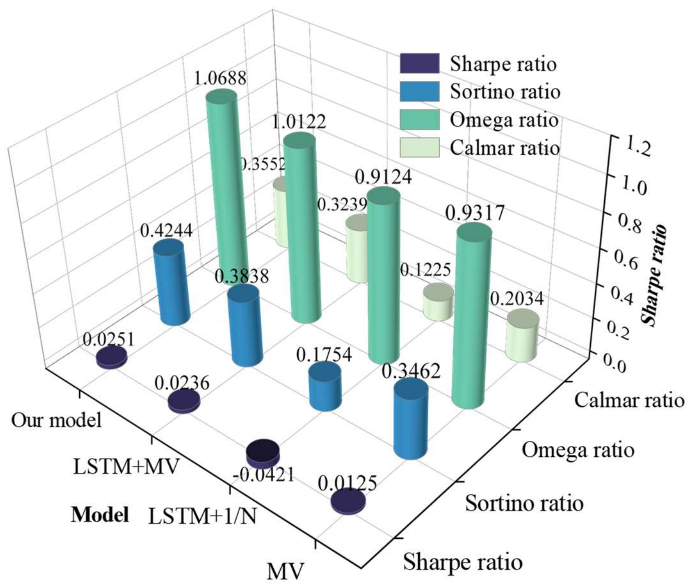

Figure 7: Evaluation of performance risk income of portfolio

III. C. 5) Overall benefit analysis

In order to present the advantages and disadvantages of the three models more intuitively, this paper shows the yield curves and cumulative yield curves of the four models through visual graphs.The yield curves of the four models are shown in Fig. 8.The yield of the MV model does not fluctuate greatly and stays in the range of -0.02~0.02. The LSTM+1/N model, on the other hand, shows very large fluctuations, and the yield fluctuations of LSTM+MV are in between. The generalized MV model with multiconditional constraints fluctuates less than the LSTM+1/N model and the LSTM+MV model, ranging from -0.024 to 0.025, and it produces a smaller value of negative returns with losses and does not have as many periods as it produces positive returns, which explains the model's optimal performance in risk-return evaluation.

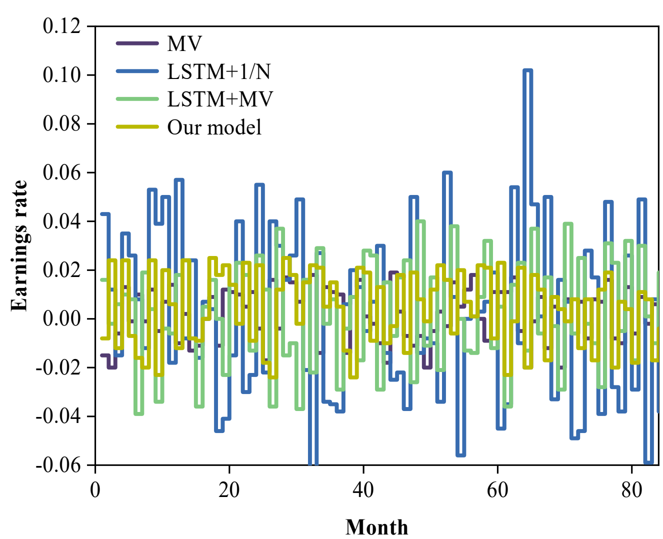

Figure 8: The yield curve of the four models

Figure 9 illustrates the cumulative return curves of the four models. The final cumulative returns of the four models are 0.146, 0.186, 0.241, and 0.359, and the generalized MV model with multiconditional constraints has a larger cumulative return than the other three strategies.

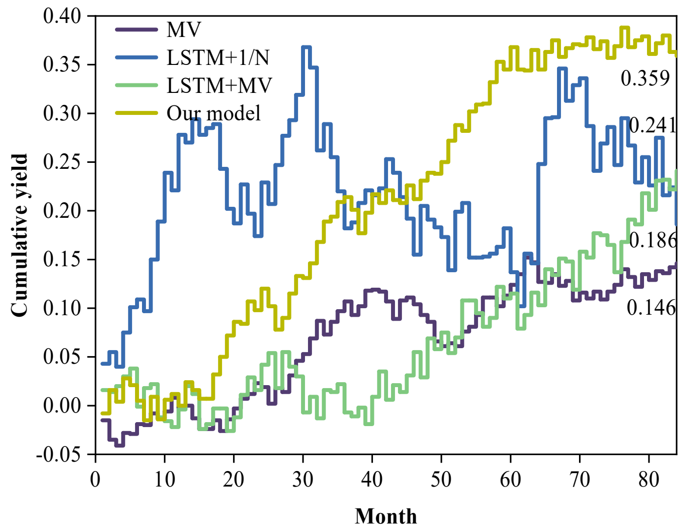

Figure 9: The cumulative yield curve of the four models

IV. Conclusion

The portfolio problem in the financial investment market has become very important for investment analysis. In order to overcome the deficiencies of the existing portfolio investment methods and risk measures in the Chinese stock market, the study introduces a variety of constraints on the basis of the mean-variance (MV) model and optimizes it using the Lagrange multiplier method. The generalized MV model with multiple constraints is explored through empirical analysis.

(1) Under the condition of bearing the same risk, the return of the portfolio composed of securities in different industries is higher than that of the portfolio composed of securities in the same industry, and the risk-to-investment ratios of the two are 0.834~1.057 and 0.823~1.038, and the number of securities selected for the optimal portfolios of the two should generally be between 11 and 18. Therefore, when choosing the securities portfolio, different industries with low correlation should be fully considered. Meanwhile, the simulation analysis verifies the validity and rationality of the generalized MV model with multi-conditional constraints.

(2) The proposed generalized MV model is outstanding in yield evaluation index, risk evaluation index and annualized risk-return index, and its Accumulated return reaches 0.2257, which is 0.0288~0.0572 higher than the comparison model.In addition, its yield curve is not highly volatile, keeping at -0.024~ 0.025, and the cumulative return finally reaches 0.359, which is 0.118~0.213 higher than the comparison model. The portfolio constructed by the generalized MV model has a better income return and risk prevention ability.

To summarize the whole paper, this paper mainly unfolds from the MV model, uses a variety of constraints to improve it, and combines the data of the Chinese securities market to carry out empirical research, and its empirical effect shows that the model has a certain reference significance for practice. However, the analysis and research work of this paper still need to be optimized, such as the Lagrange multiplier method can be improved to obtain a higher solving efficiency and solving accuracy, to help investors continuously optimize their asset allocation strategy.

References

[1] Ahmed, A. D. (2016). Integration of financial markets, financial development and growth: Is Africa different?. Journal of International Financial Markets, Institutions and Money, 42, 43-59.

[2] Busch, T., Bauer, R., & Orlitzky, M. (2016). Sustainable development and financial markets: Old paths and new avenues. Business & Society, 55(3), 303-329.

[3] Slepov, V. A., Kosov, M. E., Chalova, A. Y., Gromova, E. I., & Voronkova, E. K. (2019). Integration of the financial market sectors: factors, risks and management approaches. International Journal of Civil Engineering and Technology, 10(2), 1243.

[4] Ramaiah, K., & Soundarabai, P. B. (2024). A constrained multi-period portfolio optimization model based on quantum-inspired optimization. Multimedia Tools and Applications, 83(33), 78769-78796.

[5] Li, Y., Du, N., Song, X., Yang, X., Cui, T., Xue, N., ... & Cheah, W. P. (2024, June). Cardinality and Bounding Constrained Portfolio Optimization Using Safe Reinforcement Learning. In 2024 International Joint Conference on Neural Networks (IJCNN) (pp. 1-8). IEEE.

[6] Saunders, A., Cornett, M. M., & Erhemjamts, O. (2021). Financial institutions management: A risk management approach. McGraw-Hill.
[7] Zhang, D., Hu, M., & Ji, Q. (2020). Financial markets under the global pandemic of COVID-19. Finance research letters, 36, 101528.
[8] Lin, E. M., Sun, E. W., & Yu, M. T. (2018). Systemic risk, financial markets, and performance of financial institutions. Annals of Operations Research, 262, 579-603.
[9] Kiprop, L. F., & Tenai, J. (2017). Effect of risk identification on performance of financial institutions. International Journal of Business Strategy, 2(5), 75-87.
[10] Vesna, B. A. (2021). Challenges of financial risk management: AI applications. Management: Journal of Sustainable Business and Management Solutions in Emerging Economies, 26(3), 27-34.
[11] Tan, X., Ma, S., Wang, X., Feng, C., & Xiang, L. (2022). The impact of the COVID-19 pandemic on the global dynamic spillover of financial market risk. Frontiers in Public Health, 10, 963620.
[12] Yue, H., Liao, H., Li, D., & Chen, L. (2021). Enterprise financial risk management using information fusion technology and big data mining. Wireless Communications and Mobile Computing, 2021(1), 3835652.
[13] Angote, A. V., Malenya, A. A., & Musiega, D. (2015). Effect of enterprise financial risk management on performance in Kenya Commercial Bank, Western Region. International Journal of Business and Management Invention, 4(6), 19-40.
[14] Liagkouras, K., & Metaxiotis, K. (2018). Handling the complexities of the multi-constrained portfolio optimization problem with the support of a novel MOEA. Journal of the Operational Research Society, 69(10), 1609-1627.
[15] Hemici, M., & Zouache, D. (2023). A multi-population evolutionary algorithm for multi-objective constrained portfolio optimization problem. Artificial Intelligence Review, 56(Suppl 3), 3299-3340.
[16] Kumar, D., & Mishra, K. K. (2017). Portfolio optimization using novel co-variance guided Artificial Bee Colony algorithm. Swarm and evolutionary computation, 33, 119-130.
[17] Edirisinghe, C., Jeong, J., & Chen, J. (2021). Optimal portfolio deleveraging under market impact and margin restrictions. European Journal of Operational Research, 294(2), 746-759.
[18] de Jong, M. (2018). Portfolio optimisation in an uncertain world. Journal of Asset Management, 19(4), 216-221.
[19] Hegde, S. N., Srinivas, D. B., Rajan, M. A., Rani, S., Kataria, A., & Min, H. (2024). Multi-objective and multi constrained task scheduling framework for computational grids. Scientific Reports, 14(1), 6521.
[20] James, N., Menzies, M., & Chin, K. (2022). Economic state classification and portfolio optimisation with application to stagflationary environments. Chaos, Solitons & Fractals, 164, 112664.
[21] Turcas, F., Dumiter, F., Brezeanu, P., Farcas, P., & Coroiu, S. (2017). Practical aspects of portfolio selection and optimisation on the capital market. Economic research-Ekonomska istraživanja, 30(1), 14-30.
[22] Mazumdar, K., Zhang, D., & Guo, Y. (2020). Portfolio selection and unsystematic risk optimisation using swarm intelligence. Journal of Banking and Financial Technology, 4(1), 1-14.
[23] Loke, Z. X., Goh, S. L., Kendall, G., Abdullah, S., & Sabar, N. R. (2023). Portfolio optimization problem: a taxonomic review of solution methodologies. IEEE Access, 11, 33100-33120.
[24] Ponomareva, K., Roman, D., & Date, P. (2015). An algorithm for moment-matching scenario generation with application to financial portfolio optimisation. European Journal of Operational Research, 240(3), 678-687.
[25] Narsoo, J. (2017). Performance analysis of portfolio optimisation strategies: Evidence from the exchange market. International journal of economics and finance, 9(6), 124-132.