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Exploring the Optimal Development Path of Regional Economy in the Context of Productivity Enhancement Using Planning Algorithms

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Abstract The continuous improvement of productivity level puts forward higher requirements for regional economic development strategies and paths. For the extreme value problem with constraints involved in the regional economic development path, this paper adopts the Lagrange multiplier method to accomplish it. Firstly, the implicit function existence theorem is utilized to explore the necessary conditions for the existence of the extreme value of multivariate functions. Then the operation principle and process of Lagrange multiplier method are described in detail, and the solution steps of Lagrange multiplier method in economic optimization problems, utility maximization and cost optimization are elaborated successively. In the application research of regional economic development path, the regression coefficient of the impact of utility maximization and cost optimization on regional economic growth is 0.2483, which passes the test of 1% significance level. By applying the Lagrange multiplier method to the study of regional economy, the rapid development of regional economy is effectively promoted.

Index Terms Lagrange multiplier method, regional economic development, utility maximization, cost optimization

I. Introduction

With the in-depth development of economic globalization and the continuous advancement of technological innovation, regional economic development is facing new challenges and opportunities [1]. The traditional economic growth model has been difficult to meet the development needs of the new era, and regional productivity enhancement has become the core driving force to promote high-quality economic development [2], [3]. Regional productivity enhancement not only emphasizes the improvement of production efficiency, but also pays more attention to the enhancement of innovation ability, resource allocation efficiency and sustainable development ability [4], [5]. As a representative of contemporary advanced productivity, the new quality productivity, characterized by its high-tech, high-efficiency and high-quality, has become a key force to promote the high-quality development of regional economy [6]-[8]. The enhancement of productivity relies on the central role of scientific and technological innovation in the development process, and through the breakthrough progress of technological revolution, the optimal allocation of production factors and the leap in quality are realized [9], [10].

Regional productivity focuses on the innovative allocation of production factors, which realizes the high-quality development of the regional economy by optimizing the allocation of resources and improving the efficiency of resource allocation and total factor productivity [11], [12]. In short, the direction of productivity improvement represents the direction of future economic development. In order to comply with this trend, the relevant departments and enterprises need to continuously increase the investment in research and development, cultivate high-quality talents, optimize the industrial structure, and create a good environment conducive to innovation, so as to occupy a favorable position in the global competition [13]-[15]. In this process, the relevant departments should continue to pay attention to the international frontier dynamics and technological development trends, and actively integrate into the global value chain division of labor system, which not only helps to establish a higher level of new quality productivity system, but also helps the economy to achieve high-quality development [16]-[19]. Therefore, it is of great significance to explore the internal logic of regional economic development in the context of productivity enhancement and put forward practical development paths, which can provide theoretical support and policy recommendations for the promotion of coordinated regional economic development.

For the solution of multivariate function extreme value limited by one or more conditions, this paper chooses the implicit function existence theorem to transform the conditional extreme value into the unconditional extreme value as the premise of the promotion of Lagrange multiplier method. Then it discusses the basic principle and operation steps of Lagrange multiplier method, and transforms the utility maximization and cost minimization in economic optimization problems into mathematical models for solving. Finally, the feasibility of the Lagrange multiplier method in the utility maximization problem and cost optimization problem is tested with the case of portfolio analysis and the case of supply chain structure model, respectively. The empirical analysis of the impact of economic optimization solution based on Lagrange multiplier method on the level of regional economic development is carried out.

II. Solving and generalizing the Lagrange multiplier method

This chapter utilizes the existence theorem of implicit functions, and the necessary conditions for the existence of extremal values of multivariate functions, and discusses the basic principles and arithmetic flow of Lagrange multiplier method. Through the two realities of consumer utility maximization and cost optimization in economic optimization problems, it elaborates the specific methods of solving conditional extreme values.

II. A. Conditional extrema and the Lagrange multiplier method

An extreme value problem in which the independent variables of a function are subject to constraints is called a conditional extreme value problem.

There are two common methods for dealing with conditional extremes: one is to convert conditional extremes into unconditional extremes. The second is the Lagrange multiplier method. In fact, the essence of the Lagrange multiplier method is still to transform the conditional extreme values into unconditional extreme values. This method has been elaborated in many literatures: find the extreme value of function $z=f(x,y)$ under condition $g(x,y)=0$. First, using the implicit function existence theorem, determine the implicit function from the conditional equation, substitute back into the original function, and transform the conditional extreme value into the unconditional extreme value. Next, using the necessary conditions for the existence of the extreme value of a unitary function, if the Lagrangian function as in equation (1):

$$L(x,y,\lambda)=f(x,y)+\lambda g(x,y) \tag{1}$$

The stationary point is (x_0,y_0,λ_0), then (x_0,y_0) is the possible conditional extreme point.

II. B. Lagrange multiplier method

Let the binary functions $f(x,y)$ and $\varphi(x,y)$ have first-order continuous partial derivatives in the region D, then the problem of finding the extremum of $z=f(x,y)$ satisfying condition $\varphi(x,y)=0$ in D can be transformed into the problem of finding the unconditional extremum of the Lagrangian function as in equation (2):

$$L(x,y,\lambda)=f(x,y)+\lambda\varphi(x,y) \tag{2}$$

where λ is some constant.

Thus, the function is solved as in equation (3):

$$z=f(x,y) \tag{3}$$

The basic steps of the Lagrange multiplier method at the extremes of condition $\varphi(x,y)=0$ are:

(1) Construct the Lagrange function of equation (2) where λ is some constant.

(2) From the system of equations as in equation (4):

$$\begin{cases} L_s=f_s(x,y)+\lambda\varphi_x(x,y)=0 \\ L_y=f_s(x,y)+\lambda\varphi_y(x,y)=0 \\ L_\lambda=\varphi(x,y)=0 \end{cases} \tag{4}$$

Solve for x, y, and λ, where x and y are the possible extreme points of the desired conditional extremes.

The Lagrange multiplier method can be generalized to cases with more than two independent variables and more than one condition. The Lagrange multiplier method only gives the necessary conditions for the function to take an extreme value, so whether or not the points obtained by this method are extreme points needs to be discussed. In practice, however, it is often possible to determine whether the point sought is an extreme value point (or a point of maximum value) based on the nature of the problem itself. As in economics are specific practical problems, for example, to find the highest yield, maximum profit, etc., whether their maximum value is obvious, so the Lagrange multiplier method has a wide range of applications in economic optimization.

II. C. Application of the Lagrange multiplier method

II. C. 1) Consumer utility maximization

A consumer's utility function is known to be $U = X_1 X_2$, and the prices of two goods are $P_1 = 4$, and $P_2 = 2$ the consumer's income is $M = 80$. Now suppose the price of good 1 falls to $P_1 = 2$. Find how much the consumer's purchase of good 1 changes as a result of the substitution effect caused by the fall in the price P_1 of good 1.

Solving this problem can be transformed into two extreme value problems by first finding the magnitude of the utility at the original constant price. That is, find the function as in equation (5):

$$U\left(x_1, x_2\right) = x_1 \cdot x_2 \tag{5}$$

at the maximum of the conditional function as in equation (6):

$$x_1 \cdot P_1 + x_2 \cdot P_2 = M \tag{6}$$

Construct the Lagrangian function as in equation (7):

$$L\left(x_1, x_2, \lambda\right) = X_1 \cdot X_2 + \lambda\left(x_1 \cdot P_1 + X_2 \cdot P_2 - M\right) \tag{7}$$

From the system of equations Eq. (8):

$$\begin{cases} \dfrac{\partial f}{\partial x_1} = x_2 + \lambda P_1 = 0 \\ \dfrac{\partial f}{\partial x_2} = x_1 + \lambda P_2 = 0 \\ \dfrac{\partial f}{\partial \lambda} = x_1 \cdot P_1 + x_2 \cdot P_2 - M = 0 \end{cases} \tag{8}$$

The resulting system of equations is solved to obtain equation (9):

$$\begin{cases} x_1 = \dfrac{M}{2P_1} \\ x_2 = \dfrac{M}{2P_2} \end{cases} \tag{9}$$

When $P_1 = 4$, $P_2 = 2$, the optimal consumption bundle can be derived as equation (10):

$$\left(X_1', X_2'\right) = (10, 20) \tag{10}$$

At this point the consumer maximizes the level of utility at the given budget line as $U = x_1 \cdot x_2 = 200$. Next, find the substitution effect due to price change. When the price of a good decreases, the purchasing power of the consumer increases, therefore, to keep purchasing power unchanged it is necessary to reduce the consumer's income. That is, find the minimum value of the function as in equation (11) under the conditional function $U = x_1 \cdot x_2 = 200$:

$$f\left(x_1, x_2\right) = 2 \cdot x_1 + 2 \cdot x_2 \tag{11}$$

Construct the Lagrangian function as in equation (12):

$$F\left(x_1, x_2, \mu\right) = 2 \cdot x_1 + 2 \cdot x_2 + \mu\left(x_1 \cdot x_2 - 200\right) \tag{12}$$

Using the same method as above, find $x_1 = x_2 = 10\sqrt{2}$. So find the substitution effect due to a decrease in the price P_1 of good 1 to be $10\sqrt{2} - 10 \approx 4.14$.

II. C. 2) Cost optimization problem

Consumers are concerned about how to get maximum satisfaction with limited money and one of the points that producers are concerned about is how to optimize their costs, optimization of costs can lead to increase in profits. After the budget is fixed, how to allocate the factors of production in order to maximize the output under this constraint, the extreme value problem under the constraint, here the Lagrange multiplier method can be used to solve the conditional extreme value problem.

Suppose that the two factors of labor and capital used by a manufacturer to make a good are x_1 and x_2, respectively, that its Cobb · Douglas production function is $f\left(x_1,x_2\right)$, that the price of a unit of labor and the cost of a unit of capital are p_1 and p_2, respectively, and that the cost budget is ω. The restriction is equation (13):

$$\omega = p_1x_1 + p_2x_2 \tag{13}$$

Introduce the Lagrange multiplier λ and construct the Lagrange function as in equation (14):

$$L\left(x_1,x_2,\lambda\right) = f\left(x_1,x_2\right) + \lambda\left(p_1x_1 + p_2x_2 - \omega\right) \tag{14}$$

Then find the partial derivative of $L\left(x_1,x_2,\lambda\right)$ and the partial derivative satisfies equation (15):

$$\begin{cases} \dfrac{\partial L}{\partial x_1} = \dfrac{\partial f\left(x_1,x_2\right)}{\partial x_1} + \lambda p_1 = 0 \\ \dfrac{\partial L}{\partial x_2} = \dfrac{\partial f\left(x_1,x_2\right)}{\partial x_2} + \lambda p_2 = 0 \\ \dfrac{\partial L}{\partial \lambda} = p_1x_1 + p_2x_2 - \omega = 0 \end{cases} \tag{15}$$

where $\frac{\partial f}{\partial x_1}$, $\frac{\partial f}{\partial x_2}$ are the marginal yields of x_1, x_2 respectively, by which is meant the total increase in output from an increase of one unit of that factor of production. To find the stabilization point, it is found that when the conditions of the above equation are satisfied there is equation (16):

$$\lambda = \frac{\partial f\left(x_1,x_2\right)}{p_1\partial x_1} = \frac{\partial f\left(x_1,x_2\right)}{p_2\partial x_2} \tag{16}$$

This means that when the ratio of the marginal output of each factor of production to its price is the same and this number is the Lagrange multiplier λ, the output can be maximized and the cost of production can be optimized under the condition that the cost budget is fixed.

Similarly, this method can be extended to solve the problem of cost optimization for multiple commodities.

III. Application and testing of the Lagrange multiplier method

In this chapter, we first construct a portfolio analysis model based on the Lagrange multiplier method, compare the analysis results of this model with the probability-dependent model in multiple dimensions, and verify the superiority of the Lagrange multiplier method in the utility maximization problem. Then the functional daily necessities are taken as the research object to verify the feasibility of the Lagrange multiplier method in the cost optimization problem. Finally, the Lagrange multiplier method is applied to the research and empirical analysis of regional economic development paths, the endogeneity problem is tested to ensure the reliability of the empirical results, and the stability of the empirical evidence is tested by using the robustness test.

III. A. The utility maximization problem

III. A. 1) Analysis and selection of data

The criteria for a good or bad investment can be most intuitively reflected by the rate of return, which is high, and generally speaking, such an investment is a meaningful investment. If the rate of return is very low or even negative, the investment should be avoided. In order to verify the feasibility of this paper's model, before conducting the formal study, this paper first selected a few apparently good (meaning high yield) and bad (meaning low yield) stocks, to verify, the results show that the output is in line with the reality of the situation, proving that this paper's model is feasible. Further, the returns of eight stocks of the SSE index are randomly selected for 105 consecutive weeks from January 2017 to December 2018 (data from U-Mine data), which are PAB (stock code 000001), VKA (stock code 000002), GST (stock code 000005), CBG (stock code 000009), ST (stock code 000010), PRDA (stock code 000011), CSGA (stock code 000012) and SHAHE (stock code 000015) are used for the empirical analysis of the portfolio modeling in this paper.

III. A. 2) Comparative analysis

This section unfolds the comparison of the performance of the portfolio analysis model based on the Lagrange multiplier method with the probabilistic dependent utility model, analyzing and comparing the three dimensions of computational efficiency (runtime), portfolio return profile, and the risk measure of the portfolio. For the parameter settings of the model, there are five cases (50,100), (50,500), (50,1000), (100,500), and (500,500), where (50,100) represents 100 iterations of population size 50, and the rest of the same.

The computational efficiency of the two models is shown in Figure 1.

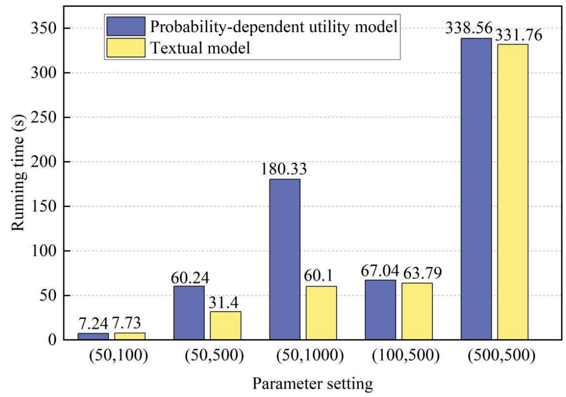

Figure 1: Comparison of computational efficiency

From the point of view of computational efficiency, the search time of the optimal solution of the model in this paper is better than that of the probability-dependent utility model in most cases, especially when the number of populations is 50 and the number of iterations is 1,000, the time gap is larger at 120.23 s. It is thus concluded that the model in this paper has better convergence performance of the model iterations relative to the probability-dependent utility model.

Based on the portfolio weights solved by the model, the comparison of the portfolio returns calculated over two years is shown in Table 1.

Table 1: Yield comparison

Parameter setting	Textual model	Probability-dependent utility model
(50,100)	-0.002076589	-0.002078326
(50,500)	-0.00216402	-0.002572228
(50,1000)	-0.000859798	-0.002157754
(100,500)	-0.000828165	-0.001881536
(500,500)	-0.001517704	-0.002001866

Under the principle of utility maximization, although the comparison effect is not very obvious, it can still be seen that the portfolio return calculated by this paper's model is slightly higher than the probability-dependent model under the same conditions, indicating that the optimization ability of this paper's model is slightly stronger. It should be noted that the portfolio returns calculated by two different utility models in two years are negative, which is mainly due to the selection of stock data, in order to make the results more general, the data selection follows the principle of randomness, randomly selected the SSE index code of the top eight stocks, and the selected data of these stocks in the negative return on the majority of cases.

In addition to the requirement that the expected return of the portfolio model is as large as possible, the relatively small risk of the portfolio model is also an aspect of the evaluation of the portfolio, and the following risk measure for the portfolio. The risk measure used in this paper is the ES model, which represents the average value shown in the tail of the return corresponding to less than the sub-quantile for a given quantile. Figure 2 shows the risk of the portfolio based on the weights obtained from the two models (with a confidence level of 95%).

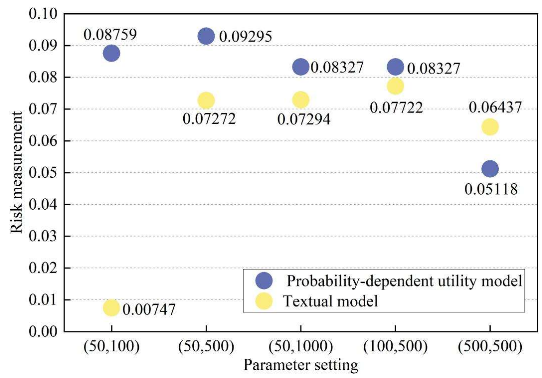

Figure 2: Comparison of risk measures

At a confidence level of 95%, the ES values calculated by the probabilistic dependence model are all larger than those calculated by the model in this paper, with a maximum difference of 0.08012, indicating that the portfolio calculated by the probabilistic dependence utility model may imply more risk.

III. B. Cost optimization problem

In this section, an efficient four-level supply chain structure model consisting of supplier-distribution center-retailer-customer is designed based on the Lagrange multiplier method for functional daily necessities such as apparel and groceries. The supply chain model considers the distribution center location, transportation type selection, transportation volume, and retailer pricing decision problems.

The initial data related to the model in this paper incorporates the actual context of the supply chain problem and is generated by computer simulation. For the supply chain cost minimization model, it is assumed that there are four suppliers, three retailers, up to four distribution centers are built, and the available transportation types are two, and the other parameters are selected in Table 2.The mathematical model of this paper is a mixed integer programming problem, which can be solved by the software matlab.

Table 2: Model initial parameter selection

Supplier s=1,…,S	Distribution center d=1,…,D	Shopkeeper r=1,…,R
O_s=700	$C_d \in [100,200]$	$\beta_r \in [0.3,0.8]$
$C_m \in [50,90]$	O_d=500	$U_r \in [1,5]$
$T_m \in [200,400]$	$V_d \in [1,3]$	$W_r \in [3,7]$
$\beta_m \in [1,6]$	$\beta_{drm} \in [1,6]$	$Q_r \in [30,120]$

Figure 3 illustrates the change in total supply chain cost as customer demand and order quantities vary between 20 and 1.2 million units.

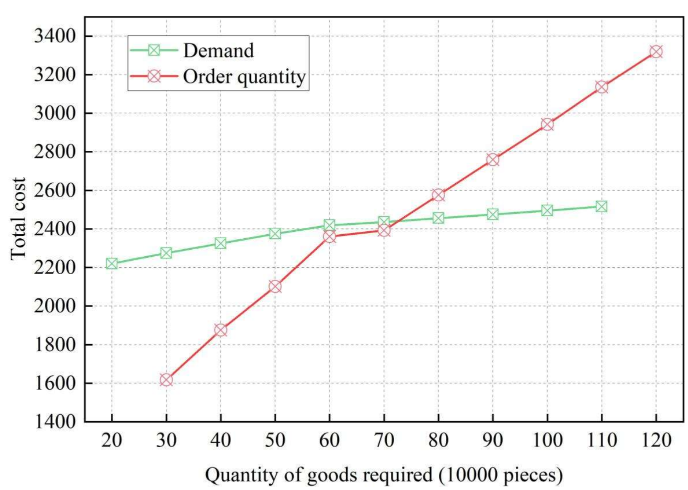

Figure 3: The change of total cost with demand and order quantity

When the order quantity remains constant, the total supply chain cost tends to increase gradually as the demand increases. When the demand is less than the order quantity, i.e., when the demand is less than 700,000 pieces, the total cost increases at a larger rate, and when the demand is greater than the order quantity, the rate of increase of the total cost starts to slow down. When the demand is constant, the total cost has the same trend with the change in ordering quantity. A comparative analysis of Figure 3 shows that the change in cost with constant demand and change in ordering quantity is greater than the change in cost with constant ordering and change in demand, which suggests that, for everyday items for which demand is determinable, how a company adjusts its own ordering quantity has a crucial role to play in reducing costs.

III. C. Impact of economic optimization on regional economic development

III. C. 1) Data sources

The basic data for the explanatory variables in the empirical analysis of this study are derived from the 2011-2020 Statistical Yearbook of China's Regional Economic Development. The basic data for the explanatory variables in the empirical analysis are derived from the Ministry of Commerce, City Statistical Yearbook and China Customs Database, which are utilized in this study to calculate the level of China's regional economic development. The basic data for the control variables and mediating variables are the data collated from the Ministry of Commerce, City Statistical Yearbook and so on. Data with missing values are filled in by linear interpolation. The descriptive statistical information of the variables is shown in Table 3, and the explanatory variables used in the empirical analysis of this section are: regional gross domestic product (GDP), regional per capita gross domestic product (PGDP), the explanatory variables are: the regional economic development level (DL), the control variables are: the informationization level (INTERNET), the level of total savings (SAVE), and the level of scientific expenditures (SAG), and the mediating variables are: Industrial Structure Upgrading (ISU), Institutional Innovation (SI), Consumption Upgrading (CU), and the observations are set to 1000 based on the regional sample size.

Table 3: Descriptive statistics of variables

Variable	Observed value	Mean value	Standard deviation	Median	Minimum value	Maximum value
GDP	1000	50100000	53100000	31500000	2800000	3870000000
PGDP	1000	72861.04	38543.26	65734.000	8581.000	468000
DL	1000	9.68	12.35	5.671	8581.000	100.000
INTERNET	1000	191.89	190.78	138.501	0.000	2089.000
SAVE	1000	0.66	0.30	0.626	0.005	2.727
SZG	1000	0.00	0.00	0.004	0.000	0.024
ISU	1000	0.08	0.02	0.069	0.000	0.079
SI	1000	12.14	2.28	12.150	5.825	19.018
CU	1000	13610.17	6580.02	12619.215	19990.547	71028.553

III. C. 2) Analysis of baseline regression results

Table 4 shows the results of the benchmark regression. In order to ensure that the regression results of this paper are real and reliable, under the premise of controlling the individual effect and point-in-time effect, each control variable is gradually introduced for regression, and the results are organized in (1) to (5) in Table 4. Where (1) is the regression result without adding any control variables, the result shows that the regression coefficient of the effect of utility maximization and cost optimization on the regional economy is 0.5793, which passes the test at 1% significance level, indicating that the effect of utility maximization and cost optimization is positive, i.e., the utility maximization and cost optimization promotes the growth of the regional economy. The results of stepwise regression after introducing each control variable are shown in (2) to (5). The results show that after the introduction of control variables, the coefficient of the impact of utility maximization and cost optimization on the regional economy is still positive and passes the test at 1% significance level, and the sign of the variable and the significance level have not changed in any way, which indicates that the control variables introduced in this paper are reasonable and effective. Meanwhile, after adding the control variables, the adjusted R-square value of each regression model is greater than 0.5, and the F-statistic value passes the test at 1% significance level, which indicates that the overall fit of each regression model is more satisfactory.

Table 4: Baseline regression result

	(1)	(2)	(3)	(4)	(5)
Variable	lnGDP	lnGDP	lnGDP	lnGDP	lnGDP
ln_DL	0.5793***	0.3231***	0.2863***	0.2312***	0.2483***
	(11.98)	(5.00)	(4.72)	(4.77)	(5.31)
ln_INTERNET		0.2344***	0.2085***	0.1726***	0.1437***
		(5.18)	(4.76)	(3.76)	(3.46)
SZG			32.5838***	33.2237***	29.8215***
			(5.94)	(5.62)	(6.05)
IRIE				0.0068***	0.0064***
				(2.88)	(3.04)
SAVE					-0.2446***
					(-4.85)
Constant term	16.2908***	15.6038***	15.6698***	15.3211***	15.6594***
	(189.60)	(102.02)	(107.96)	(108.24)	(101.39)
Urban fixation	YES	YES	YES	YES	YES
Year fixation	YES	YES	YES	YES	YES
N	999	999	999	999	999
adj.R^2	0.6705	0.6849	0.7238	0.7397	0.7729

Note:The t value in parentheses, "*, **, ***" represent significance at the significance level of 10%, 5%, and 1% respectively,the same below

From the final regression result (5), the regression coefficient of the effect of utility maximization and cost optimization on regional economic growth is 0.2483, which passes the 1% significance level test. Utility maximization and cost optimization can provide more superior resources and strategies for enterprises in the region, thus promoting the rapid increase and growth of enterprise profits. Secondly, utility maximization and cost optimization can also improve the competitiveness of enterprises and promote the development of their business by reducing costs and improving efficiency.

According to the regression results of each control variable, the regression coefficients of the level of informatization, the level of science expenditure, and the level of innovation and entrepreneurship are all significantly positive and reach statistical significance at the 1% significance level, which indicates that these control variables have a positive impact on regional economic growth. The regression coefficient of total savings level is -0.2446 and is significantly negative at 1% significance level, which is contrary to the expectation, the possible reason is that too high level of total savings may negatively affect the current economic consumption, and the lack of effective demand will lead to the excess production capacity of the society, which will make the economic efficiency of the enterprises decrease, and ultimately slow down the regional economic development.

III. C. 3) Endogeneity test

In order to mitigate the endogeneity bias due to the possible time-path dependence of economic activities, the lagged period of the explanatory variables is used as an instrumental variable for IV estimation. This approach can eliminate the bias caused by the endogeneity problem by utilizing the independence of the instrumental variables.

Table 5 shows the results of the endogeneity test. The regression coefficients of the first order lagged term instrumental variables are are all positive and pass the 1% significance level test. In addition, through the LM test and wald test, the results show that there is no endogeneity problem and weak instrumental variable problem, which means that the alternative instrumental variables are equally valid. The sign of the regression coefficients remains consistent with the previous analysis, which further proves the validity and reliability of the previous regression results.

Table 5: Endogeneity test regression results

	(1)	(2)
	IV	IV
ln_lag_DL	0.5546***	0.3081***
	(30.66)	(11.82)
ln_INTERNET		0.1046***
		(7.08)
SZG		25.6996***
		(8.48)

IRIE		0.0026**
		(2.18)
SAVE		-0.2128***
		(-8.92)
Constant term	16.3855***	16.1173***
	(508.75)	(179.86)
N	899	899
adj.R^2	0.6403	0.7049
LM	18.57	8.57
	[0.0000]	[0.0043]
Wald	135.67	30.27

III. C. 4) Robustness Tests

In order to ensure the robustness and reliability of the above empirical results, this paper uses data shrinkage with replacement variable indicators for robustness testing. Data shrinkage is done by using the Winsorize shrinkage method, by shrinking the sample data at the 1% and 99% quartiles and then doing the regression. And the replacement indicator is done by replacing PGDP with new explanatory variables for further regression analysis. The specific robustness regression results are shown in Table 6.

Table 6: Robustness test regression results

	(1)	(2)
Variable	Data indentation	Replace the explained variable
Wln_DL	0.2641***	
	(5.29)	
Wln_INTERNET	0.1317***	
	(3.18)	
WSZG	33.4725***	
	(6.32)	
WIRIE	0.0064***	
	(3.02)	
WSAVE	-0.2749***	
	(-5.77)	
ln_DL		0.1978***
		(4.91)
ln_INTERNET		0.1256***
		(3.36)
SZG		13.4467***
		(2.68)
IRIE		0.0087***
		(4.72)
SAVE		-0.2121***
		(-4.21)
Constant term	15.6968***	9.4188***
	(106.08)	(58.68)
N	999	999
adj.R^2	0.6865	0.6626

From the results of the two robustness tests, the regression results after the data shrinkage treatment show that the regression coefficient of the explanatory variables is 0.2641, which passes the 1% significance level test, and the significance level and the sign of the coefficients are basically the same as those of the previous results, which indicates that the regression results are still stable after dealing with the outliers, and are not affected by the outliers. Similarly, the results of replacing the explanatory variables also show that the regression value of utility maximization and cost optimization is 0.1978, which also passes the 1% significance level test, indicating that the results of replacing the explanatory variables show that utility maximization and cost optimization can effectively promote regional economic growth. By controlling other variables that may affect the results of the study, the regression coefficients from the empirical study of this paper are roughly the same as the previous benchmark regression results, with little change, indicating that the empirical results are relatively stable.

IV. Conclusion

For the regional economic development optimization problems arising from the increasing productivity level, this paper introduces the Lagrange multiplier method, which transforms the utility maximization and cost optimization problems in the economic optimization problems into mathematical models for solving. It is a successful attempt to apply the Lagrange multiplier method in the actual economic development and optimization problems.

Taking the utility maximization problem as the premise, the portfolio analysis model based on the Lagrange multiplier method not only has a shorter running time for solving, with a maximum difference of 120.23s, but also has a stronger optimization ability and lower embedded risk than similar models. In addition, in the empirical analysis of the impact on regional economic development, the regression coefficient of the Lagrange multiplier method dominated by utility maximization and cost optimization on regional economic growth is 0.2483, which passes the test of significance level of 1%.

References

[1] Yang, Z., Shao, S., Xu, L., & Yang, L. (2022). Can regional development plans promote economic growth? City-level evidence from China. Socio-Economic Planning Sciences, 83, 101212.

[2] Zhang, J., Teng, T., Wang, T., & Zeng, G. (2024). The Path Innovation of Regional Economic Growth: Data Analysis Based on China. Journal of the Knowledge Economy, 1-24.

[3] Zhou, B., Wang, N., Zhang, Z., Liu, W., Lu, W., Xu, R., & Li, L. (2022). Research on the Spatial-temporal differentiation and path analysis of China's provincial regions' high-quality economic development. Sustainability, 14(10), 6348.

[4] Luo, K., Lee, C. C., & Zhuo, C. (2024). A pathway to coordinated regional development: Energy utilization efficiency and green development-Evidence from China's major national strategic zones. Energy Economics, 131, 107402.

[5] Shen, W., Xia, W., & Li, S. (2022). Dynamic coupling trajectory and spatial-temporal characteristics of high-quality economic development and the digital economy. Sustainability, 14(8), 4543.

[6] Hua, X., Lv, H., & Jin, X. (2021). Research on high-quality development efficiency and total factor productivity of regional economies in China. Sustainability, 13(15), 8287.

[7] Liu, Y., & He, Z. (2024). Synergistic industrial agglomeration, new quality productive forces and high-quality development of the manufacturing industry. International Review of Economics & Finance, 94, 103373.

[8] Li, L., & Liu, Z. (2024). Sustainable Evolution of China's Provincial New Quality Productivity Based on Three Dimensions of Multi-Period Development and Combination Weights. Sustainability, 16(24), 11259.

[9] Ding, C., Liu, C., Zheng, C., & Li, F. (2021). Digital economy, technological innovation and high-quality economic development: Based on spatial effect and mediation effect. Sustainability, 14(1), 216.

[10] Li, L., Arip, M. A., & Hong, P. C. (2025). Spatiotemporal evolution patterns of the coupling of carbon productivity and high-quality economic development in China. Scientific Reports, 15(1), 9114.

[11] Spithoven, A., & Merlevede, B. (2023). The productivity impact of R&D and FDI spillovers: characterising regional path development. The Journal of Technology Transfer, 48(2), 560-590.

[12] Hu, Y., Liu, J., Zhang, S., Liu, Y., Xu, H., & Liu, P. (2024). New mechanisms for increasing agricultural total factor productivity: Analysis of the regional effects of the digital economy. Economic Analysis and Policy, 83, 766-785.

[13] Wang, S., Lu, B., & Yin, K. (2021). Financial development, productivity, and high-quality development of the marine economy. Marine Policy, 130, 104553.

[14] Pan, W., Xie, T., Wang, Z., & Ma, L. (2022). Digital economy: An innovation driver for total factor productivity. Journal of business research, 139, 303-311.

[15] Yang, S., & Liu, W. (2024, October). Research on High-quality Development Strategy of New Quality Productivity in Digital Era. In 4th International Conference on Internet Finance and Digital Economy (ICIFDE 2024) (pp. 51-61). Atlantis Press.

[16] Malecki, E. J. (2018). Technological innovation and paths to regional economic growth. Growth policy in the age of high technology, 97-126.

[17] Talebzadehhosseini, S., & Garibay, I. (2022). The interaction effects of technological innovation and path-dependent economic growth on countries overall green growth performance. Journal of Cleaner Production, 333, 130134.

[18] Asheim, B. T. (2019). Smart specialisation, innovation policy and regional innovation systems: what about new path development in less innovative regions?. Innovation: The European Journal of Social Science Research, 32(1), 8-25.

[19] Yuan, X. M., Wei, F. L., Li, H., & An, Y. (2022). Analysis of the Coupling Coordination and Spatiotemporal Evolution of High-Tech Industrial Technological Innovation and Regional Economic Development. Complexity, 2022(1), 3076336.