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Product visual communication style innovation inspired by fractal geometry

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Abstract This paper realizes the productized application of fractal geometric aesthetics through parametric modeling and fractal algorithm. Firstly, we systematically analyze the mathematical principles of fractal generation such as Hilbert curve, Piano curve, etc., and construct the mapping relationship between fractal forms and product decoration patterns. A Grasshopper-based fractal parametric design framework is proposed to realize the dynamic evolution and functional adaptation from two-dimensional fractal patterns to three-dimensional product forms. The fractal algorithm is developed, combining recursive algorithm and iterative algorithm optimization with grayscale symbiotic matrix feature screening mechanism to quantitatively evaluate the adaptability of fractal texture in the visual representation of products. The fractal algorithm in this paper generates images with higher accuracy, with an average accuracy of 86.77%, while the average satisfaction is improved by 12.35% over the traditional algorithm.

Index Terms fractal geometry, parametric modeling, Grasshopper, grayscale covariance matrix

I. Introduction

In today's competitive market environment, product design has experienced rapid development, and innovations continue to emerge, pushing the discipline of product design to a whole new level [1], [2]. With the development of modern technology, product design innovation has become an important driving force to promote the development of the industry. Product design innovation has become a key factor for enterprises to stand out and win market share. Successful product design innovation can not only meet the needs of consumers and promote consumption, but also bring great commercial value to enterprises [3]-[5]. In product design, its visual communication style aims to attract the attention of the audience, convey specific information, create emotional resonance, and establish a specific brand or message image [6], [7]. Visual communication is an art and design practice that conveys information, ideas, or emotions through the organization and presentation of visual elements such as images, graphics, colors, and typography [8]. It is a process that draws on visual language and design principles through the organic combination of visual elements in order to effectively convey and communicate.

Euclidean geometry studies regular geometries that are continuous and differentiable everywhere, however, the shapes of materials in nature such as mountains, coastlines, trees, etc. are irregular and complex [9], [10]. The emergence of fractal theory makes it possible to explore the regularity behind complex phenomena, which explains the essential relationship between the local and the whole through the chaotic appearance and irregular shape [11], [12]. Product design and manufacturing implies the metaphor of life, and all its principles are embedded in the living body. Applying fractal geometry theory to product design can make products closer to life and nature.

This paper first analyzes the mathematical nature of fractal curves and explores its geometric translation logic in decorative pattern design. A fractal parametric design method is constructed, and recursive and iterative algorithms are applied to test the feasibility of two-dimensional and three-dimensional design in product form. Combined with gray scale feature analysis, quantify the adaptive efficacy of fractal texture in product visual communication. Design experiments to analyze the influence of the number of iterations on the fractal algorithm, and take four fractal images to be stitched together as an example to illustrate the image feature extraction process. Compare and analyze the algorithm of this paper with the traditional style migration algorithm to evaluate the superiority of the fractal algorithm.

II. Product design based on fractal theory

II. A. Definition of a fractal curve

Definition of a space-filling curve: a map $f: I \to E^n (n \geq 2)$ is continuous and $f(I)$ has a positive PeanoJordan measure. Then $f(I)$ is called a space-filling curve, where E^n denotes an n-dimensional Euclidean space.

Definition of Hilbert curve: assuming that I and Q are the intervals $[0,1]$ and the squares $[0,1]\times[0,1]$, respectively, the Hilbert curve is generated as follows:

$$\mathrm{H}: t \in [0,1] \mapsto \mathrm{H}(t) \in [0,1]\times[0,1], t = 0.q_1 q_2 \cdots, 0 \le q_j \le 3 \tag{1}$$

$$\mathrm{H}(t) = \begin{pmatrix} \mathrm{R}e \\ \mathrm{I}m \end{pmatrix} \lim_{n\to\infty} T_{q_1} T_{q_2} \cdots T_{q_n} \mathrm{Q} \tag{2}$$

where t is denoted by a quaternion. Define $\{T_i \mid 0 \le i \le 3\}$ as follows:

$$T_i = \frac{1}{2} H_i z + h_i, 0 \le i \le 3 \tag{3}$$

Continue by introducing the following expressions $H_0 z = \bar{z} i$, $H_1 z = z$, $H_2 z = z$, $H_3 z = -\bar{z} i$; $h_0 = 0$, $h_1 = \frac{i}{2}$, $h_2 = \frac{1+i}{2}$, and $h_3 = \frac{2+i}{2}$. where consider the complex $z \in$ □ as $(\mathrm{R}e(z), \mathrm{I}m(z)) \in \mathrm{Q}$. Put these four transformations $T_i \mid 0 \le i \le 3$ corresponding to different geometric deformations, respectively. Taking the transformation T_0 as an example, the original Q is first scaled down towards the origin by $\frac{1}{2}$, then multiplied by -1 to reflect it on the imaginary axis, and multiplied by the imaginary number i to rotate the square by 90 degrees. During the generation of the Hilbert curve, the subsquare shrinks to a point, which indicates that $\mathrm{H}(t)$ is a point in □^{2}. The nth approximation of the Hilbert curve is constructed by the nth iteration, denoted $\mathrm{H}(n)$:

$$\begin{aligned} \mathrm{H}_n(0.q_1 q_2 \cdots q_n) &= \begin{pmatrix} \mathrm{R}e \\ \mathrm{I}m \end{pmatrix} \sum_{j=1}^{n} \frac{1}{2^j} H_{q_0} H_{q_1} H_{q_2} \cdots H_{q_{j-1}} H_{q_j} \\ &= \sum_{j=1}^{n} \frac{1}{2^j} (-1)^{e_{0j}} \operatorname{sgn}(q_j) \begin{pmatrix} (1-d_j) q_j - 1 \\ 1 - d_j q_j \end{pmatrix} \end{aligned} \tag{4}$$

$$\operatorname{sgn}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & x = 0 \end{cases} \tag{5}$$

$$e_{kj} = \#\left(k\text{” } preceding\ q_j \right) \bmod 2 \tag{6}$$

$$d_j = e_{0j} + e_{3j} \bmod 2 \tag{7}$$

where $\operatorname{sgn}(x)$ is the sign function, # is the counting function, and $k \in \{0,3\}$. The exit point of each sub-square of the Hilbert curve coincides with the point of entry into the next sub-square.

Definition of a zigzag curve: This chapter examines a Reshape curve on an image of size $H \times W$, assuming that H and W are both equal to 1 and are equally divided into 2^n parts, given a real number $t \in [0,1]$, which can be expressed in finite-length quaternionic form as: $t = 0.q_1 q_2 \cdots q_n$, with the symbol R defined as follows:

$$\mathrm{R}: 0.q_1 q_2 \cdots q_n \to \begin{pmatrix} \left(\sum_{k=1}^{n} q_k 4^{n-k} \% 2^n \right) * \frac{1}{2^n} + \frac{1}{2^{n+1}} \\ \left\lfloor \frac{\sum_{k=1}^{n} q_k 4^{n-k}}{2} \right\rfloor * \frac{1}{2^n} + \frac{1}{2^{n+1}} \end{pmatrix} \tag{8}$$

Definition of the Piano curve: the generation process of the Piano curve is similar to that of the Hilbert curve. In this paper, we first denote the nth approximation of the Peano curve as $\mathrm{P}_n : t \in \mathrm{I} \mapsto \mathrm{P}_n(t) \in \mathrm{Q}$, and the Peano curve is the limit of P_n when n tends to infinity. The binary to decimal conversion function is denoted by B, and the Peano curve is generated as follows:

$$\mathrm{P}_n\left(\frac{\mathrm{B}(q_1q_2\cdots q_n)}{2^n-1}\right)=\begin{pmatrix}\dfrac{\mathrm{B}(q_1q_3\cdots q_n)}{2^{n/2}}\\ \dfrac{\mathrm{B}(q_2q_4\cdots q_{n-1})}{2^{n/2}}\end{pmatrix} \square\ p\in\mathrm{Q} \tag{9}$$

$$\mathrm{P}_n(t)=p_0*(1-s)+p_1*s, t\in[\mathrm{P}_n^{-1}(p_0),\mathrm{P}_n^{-1}(p_1)] \tag{10}$$

where $s=\dfrac{t-\mathrm{P}_n^{-1}(p_0)}{\mathrm{P}_n^{-1}(p_1)-\mathrm{P}_n^{-1}(p_0)}\in[0,1]$, $q_i\in\{0,1\}$.

II. B.Decorative patterns in product design

Decorative patterns play an important role in product design, it renders the artistic atmosphere of the design object, improves the aesthetic character of the product; strengthens, reminds and guides the audience's sight. Decorative patterns play an important role in product design, rendering the artistic atmosphere of the design object, improving the aesthetic character of the product, and strengthening, reminding and guiding the audience's vision. Decorative patterns in product modeling are divided into the following categories in a general sense from the point of view of material extraction: plant patterns, animal patterns, landscape patterns, character patterns, geometric patterns and so on.

In a general sense, geometric shape refers to the geometric system established by the ancient Greek mathematician Euclid's organization, such as: square, trapezoid, circle, pentagram, triangle, hexagon, fan, bow, circle, rectangle, cylinder, platform, prism, prism, cone, prism, and so on, and a few common geometric shapes are shown in Figure 1.

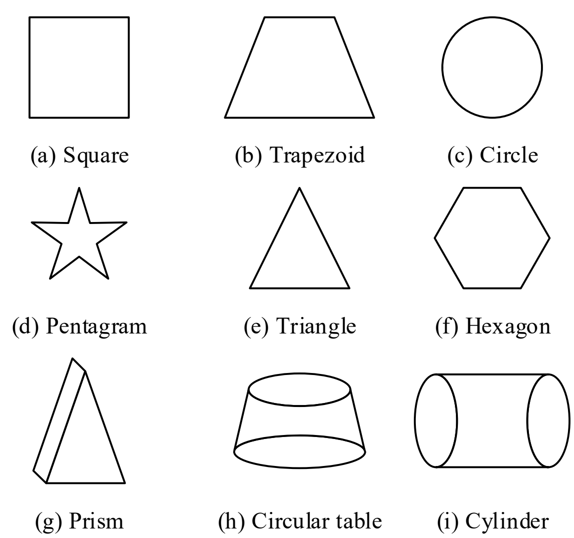

Figure 1: Several common basic geometries

Geometric pattern is the use of point and line surface combination of popular aesthetic patterns, generally categorized as abstract graphics, in the actual design there are also geometric and natural forms combined to form a semi-abstract graphics.

The source of the geometric patterns often used in design is mainly from the reality of animals, plants and other figurative objects gradually evolved, simplified, abstracted into a geometric pattern, such as: a variety of straight lines, curves, vortex patterns, triangles, serrated patterns and so on.

Geometric decorative graphics in product design use point shape change, line shape change and surface shape change to form the graphics with no direct meaning between the product function but have indirect connection. Its free expression, diverse forms, strong sense of the times, to create more space for consumers to associate.

II. C.Parametric design of product form based on fractal theory

With the development of parametric design and computational aesthetics, fractal geometry, due to its self-similarity and infinite recurrence, has gradually become an innovative tool to break through the limitations of traditional product design forms. In the overall process of parametric design, the Grasshopper plug-in is used to establish a parametric design model and adjust the basic parameters of the design. After determining the logical structure of the product data, according to the corresponding parameter modification to achieve a more advanced and complicated logic modeling instructions, and this rationalization of the logic into a visual visual whole, to generate a dynamic morphology analysis, to observe the changes in the product morphology, each step can be modified according to the needs of the product design, fractal parameters, adjust the details of the design, and ultimately find out the optimal form of the product after the optimization of the details. This design process takes parametric design as the core to develop the product form design, which improves the design efficiency, shortens the R&D cycle, and ensures the effectiveness of the design. The design flow of parameterized product form is shown in Figure 2. Two-dimensional and three-dimensional two-step realization is carried out.

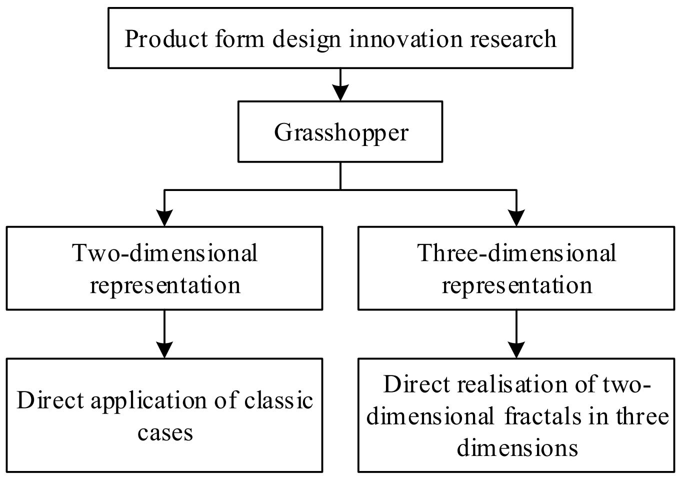

Figure 2: Parametric design process

Two-dimensional expression: Taking product form and technological innovation as the basic carrier, the classical image of fractal is directly applied to product form design, i.e., through the basic design means of decomposition and reorganization, the fractal graphic is directly applied to the product modeling unit body, and through the regular combination of the units, it is made into a whole. Adopting fractal algorithm model for two-dimensional product appearance expression, the generated multiple two-dimensional fractal patterns are applied to other details processing of product form design, which increases the external visual effect and the innovative ideas of three-dimensional realization.

Three-dimensional expression: directly realize the three-dimensionalization of two-dimensional fractal, and innovate on the basis of the original fractal, refine, deform and other abstract generalization of the features of the fractal image, and apply them to the construction and combination of the product form, so as to achieve to satisfy the functional requirements of the product and the overall law of the design. Adoption of fractal algorithm for product modeling optimization design maximizes the innovative characteristics of the product, realizes the three-dimensional diversified ethnic grouping system, and thus promotes the development of innovative design of product form.

In this paper, from the perspective of fractal, according to the design process of product form parameterization, the recursive algorithm and iterative algorithm of fractal algorithm are used to check the feasibility of two-dimensional and three-dimensional in product form design. And combined with the plug-in tool Grasshopper, it is parameterized through the parametric method to provide basic support for fractal theory and algorithms in the innovative design of product form, and at the same time, it also enriches the external manifestation of the product form design and generates the community system of three-dimensional innovative form of the product.

III. Analysis of the application of visual communication style in product design based on fractal geometry

III. A. Application process of fractal algorithm

The experimental images are collected from three brands' product databases and Fractal Foundation open source library, and a benchmark dataset is constructed, which contains 1000 sets of natural fractal patterns and artificially designed patterns. The original images are pre-processed with denoising, grayscale normalization and edge enhancement to ensure that the morphological features and color distribution of the dataset meet the subsequent analysis requirements.

III. A. 1) Effect of number of iterations on fractal algorithms

In order to explore the effect of the number of iterations on the fractal algorithm, the images are generated by setting different number of iterations, and a reasonable number of iterations is determined based on the effect of the generated images and the total loss value. Set the ratio of content weights to style weights and the value of variable weights. Different images are generated by setting different iteration times, and the relationship between the number of iterations and the total loss value in 300 iterations is shown in Figure 3. It can be seen that the total loss value decreases with the increase of the number of iterations, in the first 20 times the total loss value decreases sharply especially in the first 5 iterations, when the number of iterations reaches about 180 times, the total loss value tends to be stabilized, and the final number of iterations can be set to 180 times.

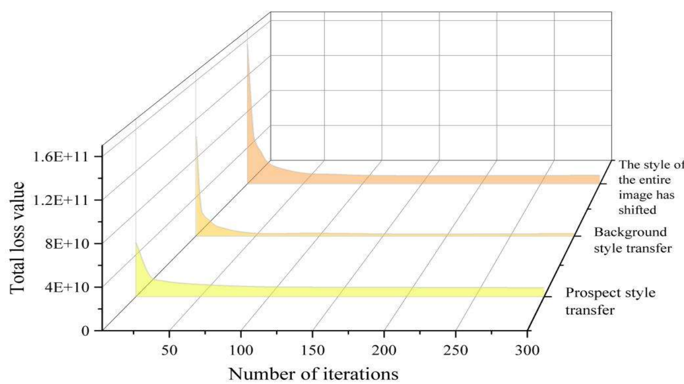

Figure 3: Relationship between the number of iterations and the total loss value

III. A. 2) Image Feature Extraction

Due to the different resolutions of the images in the fractal atlas, which leads to the problem of excessive size of the splicing effect, the fractal graphics should be scaled to the specified resolution interval first. In this paper, the rectangular window round search mechanism is used to extract image features, and the texture features of fractal graphs are extracted through the grayscale covariance matrix, to determine whether the texture features meet the splicing requirements and to choose the appropriate splicing method.

Taking four fractal maps (numbered A~D) to be spliced as an example, the rectangle wheel-seeking mechanism is used to extract the target region for them. Rectangle wheel search mechanism is to use a rectangle window to traverse the whole picture according to the set step size, narrow down the global picture to the window field of view, and then operate on the pixel points in the rectangle window. The default length and width of the rectangular window are set to be 1/4 of the size of the original image.The probability distribution of the occurrence of the grayscale levels of the four fractal images to be spliced is shown in Fig. 4. The gray level histogram A shows a significant single-peak concentration distribution, and its gray level probability reaches the peak (0.021) in the interval of 20-30, followed by an exponential decay trend of the probability tends to be close to zero. Histogram D presents a multi-peak asymptotic distribution in the 0-150 interval, in which a sub-peak (0.013) occurs at the 50-60 level of gray scale, and its probability distribution is more discrete than that of the A graph.

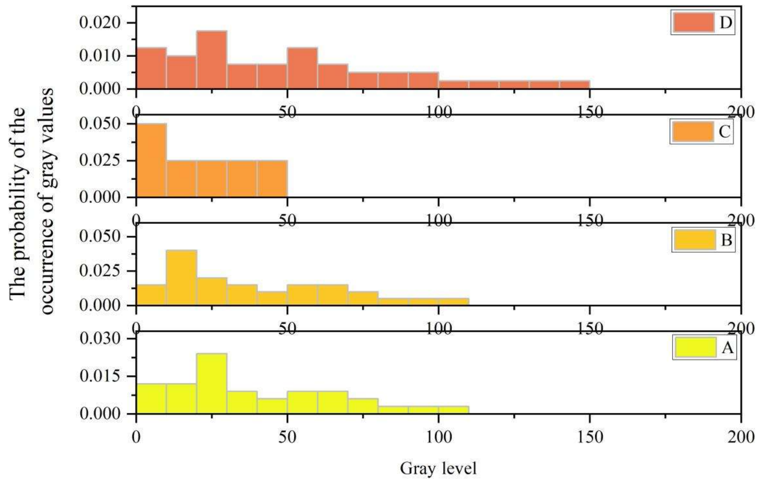

Figure 4: Distribution of occurrence probability of gray level

OTSU binarization was performed on the fractal map using gray scale probability to obtain maximum inter-class variance, and the number of pixel points within the rectangle was counted. If the percentage of non-zero pixel points within the rectangle reaches 1/5, it indicates that the rectangle is of appropriate size, and the coordinates of the starting point of the rectangle containing the edge pixels are recorded. If the proportion is less than 1/5, the rectangle is larger, and the rectangle length and width are reduced in turn. If the proportion of non-zero pixels is still less than 1/5 after the rectangle length and width are reduced to 1/8 of the image, the target region is judged to be too small for extraction.

After determining the region of interest (ROI), the ROI region is extracted and used as the base texture for the next texture analysis. Texture features are correlations in the gray space of the picture, and the gray scale co-production matrix is an effective method for extracting the texture information of the picture. In this paper, for the grayscale symbiosis matrix set up 4 directions of 0 degrees, 60 degrees, 120 degrees, 150 degrees, the grayscale is divided into 16 levels, and the average of 5 features commonly used to extract texture feature information in 4 directions: angular second-order moments, entropy, contrast, inverse disparity, and correlation. Combined with the gray scale feature requirements of the spliced texture, the uniform metric formula for the four-sided gray scale distribution adapted to the fractal atlas is proposed H. The correlation formula is as follows:

$$Asm = \sum_i \sum_j P(i,j)^2 \tag{11}$$

$$Ent = \sum_i \sum_j P(i,j) \log_2 P(i,j) \tag{12}$$

$$H = w_1 Asm + w_2 Ent, ASM > 0.2, Ent < 4 \tag{13}$$

In the above equation, $P(i,j)$ denotes the different gray scale co-production matrix elements in the four directions, $w_1 = 5$ (angular second-order moments) is a measure of the degree of gray scale variation and stability of the image texture, with larger values representing strong gray scale uniformity and high degree of texture regularity, and the Asm value is positively correlated with gray scale uniformity, so w_1 is taken as positive. Ent (entropy) is a measure of the complexity of the gray scale distribution, the smaller the value indicates that the texture complexity is lower, Ent value is negatively correlated with the texture complexity, so w_2 is taken as negative. Formula H (13) is a combination of two gray-scale distribution characteristics, used to screen the four sides of the selection of gray uniformity and low complexity of the fractal graph, the resulting $H > 1$ when the conditions are considered to be met.

The extracted features of the gray scale covariance matrix of the example image are shown in Table 1. It can be seen that the H values of the images to be spliced A, B and D are all greater than 1, indicating that the texture of their four sides is simple and uniform, which is suitable for the splicing operation. And the H value of image C to be spliced is only 0.803, which does not meet the splicing conditions.

Table 1: Features extracted from gray level co-occurrence matrix

	ASM	ENT	H
A	0.736	0.801	4.024
B	0.501	1.084	2.497
C	0.208	1.738	0.803
D	0.339	2.397	1.186

III. B. Application effects

In order to verify the practical efficacy of fractal algorithms in product design, this study constructs a multi-dimensional evaluation system, and analyzes the effect through the dual validation framework of objective accuracy index and subjective satisfaction evaluation. The subjective satisfaction evaluation part invites 32 industrial designers as subjects to rate the generated patterns. The traditional style migration algorithm is chosen as the control group, and the same fractal pattern material is selected to generate design patterns, and three groups of design patterns are analyzed. As shown in Table 2, the fractal algorithm in this paper generates images with higher accuracy, with an average accuracy of 86.77%, and at the same time, the average satisfaction is 12.35% higher than the traditional algorithm.

Table 2: Comparison Results of Accuracy and satisfaction

Model	The generated image	Accuracy/%	Satisfaction/%
Traditional	P1	65.28	74.42
The proposed		85.73	89.28
Traditional	P2	70.14	79.13
The proposed		88.36	90.26
Traditional	P3	71.09	81.25
The proposed		86.22	92.33

IV. Conclusion

In this paper, a product design method based on fractal theory is designed, and its design process and practical application effect are elaborated through experiments.

The total loss value of the fractal algorithm decreases with the increase of the number of iterations, the total loss value decreases sharply in the first 20 times especially in the first 5 iterations, and when the number of iterations reaches about 180 times, the total loss value tends to be stabilized, and the final number of iterations can be set at 180 times. Taking the four fractal maps to be spliced (numbered A~D) as an example, the grayscale histogram A shows a significant single-peak centralized distribution characteristic, and its grayscale probability peaks in the 20-30 interval (0.021), followed by an exponential decay trend of the probability tends to zero. Histogram D presents a multi-peak asymptotic distribution in the 0-150 interval, in which a sub-peak (0.013) occurs at the 50-60 level of gray scale, and its probability distribution is more discrete than that of the A graph. The H-values of the images to be spliced A, B and D are all greater than 1, indicating that their four-sided texture is simple and uniform, which is suitable for splicing operation. While the H value of image C to be spliced is only 0.803, which does not meet the condition of splicing.

The objective and subjective evaluation results show that the fractal algorithm in this paper generates images with higher accuracy, with an average accuracy of 86.77%, while the average satisfaction is 12.35% higher than the traditional algorithm.

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